Algebra – Outcome 1

2001

1.

A5. (a) Obtain partial fractions for $\dfrac{x}{x^2-1}, x>1$. **(2)**

A6. Expand $\left(x^2-\dfrac{2}{x}\right)^4, x\neq 0$ and simplify as far as possible. **(5)**

2002

A8. Express $\dfrac{x^2}{(x+1)^2}$ in the form $A+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}, (x\neq -1)$, stating the values of the constants A, B and C. **(3)**

2004

2. Obtain the binomial expansion of $\left(a^2-3\right)^4$. **(3)**

5. Express $\dfrac{1}{x^2-x-6}$ in partial fractions. **(2)**

2005

13. Express $\dfrac{1}{x^3+x}$ in partial fractions. **(4)**

2007

1. Express the binomial expansion of $\left(x-\dfrac{2}{x}\right)^4$ in the form $ax^4+bx^2+c+\dfrac{d}{x^2}+\dfrac{e}{x^4}$ for integers a, b, c, d and e. **(4)**

4. Express $\dfrac{2x^2-9x-6}{x\left(x^2-x-6\right)}$ in partial fractions. **(3)**

2008

4. Express $\dfrac{12x^2+20}{x\left(x^2+5\right)}$ in partial fractions. **(3)**

8. Write down and simplify the general term in the expansion of $\left(x^2+\dfrac{1}{x}\right)^{10}$. **(3)**
Hence, or otherwise, obtain the term in x^{14}. **(2)**

2009

8. (a) Write down the binomial expansion of $(1+x)^5$. **(1)**
(b) Hence show that $(0.9)^5$ is 0.59049. **(2)**

14. Express $\dfrac{x^2+6x-4}{(x+2)^2(x-4)}$ in partial fractions. **(4)**

2011

Express $\dfrac{13-x}{x^2+4x-5}$ in partial fractions **(2)**

Use the binomial theorem to expand $\left(\dfrac{1}{2}x-3\right)^4$ and simplify your answer. **(3)**

Answers

2001

$$\frac{x}{x^2-1}=\frac{1}{2(x+1)}+\frac{1}{2(x-1)}$$

$$x^8-8x^5+24x^2-\frac{32}{x}+\frac{16}{x^4}$$

2002

$$y=1-\frac{2}{x+1}+\frac{1}{(x+1)^2}$$

2004

$$a^8-12a^6+54a^4-108a^2+81$$

$$\frac{1}{x^2-x-6}=\frac{1}{5(x-3)}-\frac{1}{5(x+2)}$$

2005

$$\frac{1}{x^3+x}=\frac{1}{x}-\frac{x}{x^2+1}$$

2007

$$x^4-8x^2+24-\frac{32}{x^2}+\frac{16}{x^4}$$

$$\frac{1}{x}+\frac{2}{x+2}-\frac{1}{x-3}$$

2008

$$\frac{4}{x}+\frac{8x}{x^2+5}$$

$$\binom{10}{r}x^{20-3r}$$

$$45x^{14}$$

2009

$$1+5x+10x^2+10x^3+5x^4+x^5$$

$$1-0.5+0.1-0.01+0.0005-0.00001=0.59049$$

$$\frac{2}{(x+2)^2}+\frac{1}{x-4}$$

2011

$$\frac{2}{x-1}-\frac{3}{x+5}$$

$$\frac{1}{16}x^4-\frac{3}{2}x^3+\frac{27}{2}x^2-54x+81$$

2001

A2. Differentiate with respect to x

(a) $f(x)=(2+x)\tan^{-1}\sqrt{x-1}, x>1$ (b) $g(x)=e^{\cot 2x}, 0<x<\frac{\pi}{2}$. **(4) (2)**

2002

A4. (a) Given that $f(x)=\sqrt{x}e^{-x}, x\geq 0$, obtain and simplify $f'(x)$. **(4)**

2003

A1. Given $f(x)=x(1+x)^{10}$, obtain $f'(x)$ and simplify your answer. 3 marks

2004

1. a) Given $f(x)=\cos^2 xe^{\tan x}, -\frac{\pi}{2}<x<\frac{\pi}{2}$, obtain $f'(x)$ and evaluate $f'\left(\frac{\pi}{4}\right)$. 3, 1 marks

b) Differentiate $g(x)=\frac{\tan^{-1}2x}{1+4x^2}$. 3 marks

2005

1. a) Given $f(x)=x^3\tan 2x$, where $0<x<\frac{\pi}{4}$, obtain $f'(x)$. 3 marks

b) For $y=\frac{1+x^2}{1+x}$, where $x\neq -1$, determine $\frac{dy}{dx}$ in simplified form. 3 marks

15. a) Given $f(x)=\sqrt{\sin x}$, where $0<x<\pi$, obtain $f'(x)$. 1 mark

b) If, in general, $f(x)=\sqrt{g(x)}$, where g(x) > 0, show that $f'(x)=\frac{g'(x)}{k\sqrt{g(x)}}$, stating the value of k. 2 marks

2006

2. Differentiate, simplifying your answers:

a) $2\tan^{-1}\sqrt{1+x}$, where $x>-1$; 3 marks

b) $\frac{1+\ln x}{3x}$, where $x>0$. 3 marks

2007

2. Obtain the derivative of each of the following functions:

(a) $f(x)=\exp(\sin 2x)$; 3 marks

2008

9. Write down the derivative of tan x. 1 mark

Show that $1+\tan^2 x=\sec^2 x$. 1 mark

2008

15. Let $f(x) = \dfrac{x}{\ln x}$ for $x > 1$.

(a) Derive expressions for $f'(x)$ and $f''(x)$, simplifying your answers. 2, 2 marks

(b) Obtain the coordinates and nature of the stationary point of the curve $y = f(x)$. 3 marks

(c) Obtain the coordinates of the point of infexion. 2 marks

2009

1. (a) Given $f(x) = (x+1)(x-2)^3$ obtain the values of x for which $f'(x) = 0$. 3 marks

2010

1. Differentiate the following functions. (a) $f(x) = e^x \sin x^2$. (b) $g(x) = \dfrac{x^3}{(1+\tan x)}$ 3 marks 3 marks

2011

A curve is defined by the equation $y = \dfrac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}$ for $x < 1$.

Calculate the gradient of the curve when $x = 0$. 4 marks

Advanced Higher Topic Past Papers – Differentiation *Unit 1 Outcome 2*

2001

$\tan^{-1}\sqrt{x+1}+\dfrac{2+x}{2x\sqrt{x-1}}$ $g'(x)=-2e^{\cot 2x}\operatorname{cosec}^2 2x$

2002

$f'(x)=\dfrac{1}{2\sqrt{x}}e^{-x}(1-2x)$

2003

$f'(x)=(1+11x)(1+x)^9$

2004

$f'(x)=(1-\sin 2x)e^{\tan x}$; $f'\left(\dfrac{\pi}{4}\right)=0$ **(4)** $g'(x)=\dfrac{2-8x\tan^{-1}2x}{(1+4x^2)^2}$ **(3)**

2005

$f'(x)=3x^2\tan 2x+x^3(2\sec^2 2x)$ **(3)** $\dfrac{dy}{dx}=\dfrac{x^2+2x-1}{(1+x)^2}$ or $1-\dfrac{2}{(1+x)^2}$ or $\dfrac{2x}{(1+x)}-\dfrac{1+x^2}{(1+x)^2}$ **(3)**

$f'(x)=\dfrac{1}{2}\dfrac{\cos x}{(\sin x)^{1/2}}$ **(1)** $k=2$; **(2)**

2006

$f'(x)=\dfrac{1}{(2+x)\sqrt{1+x}}$ **(3)** $f'(x)=\dfrac{-\ln x}{3x^2}$ **(3)**

2007

$f'(x)=2\cos 2x\exp(\sin 2x)$

2008

9.a) $\sec^2 x$ b) Proof

15.a) $\dfrac{2-\ln x}{x(\ln x)^3}$ **(4)** b) (e, e) Min **(3)** c) $x=e^2\Rightarrow y=\dfrac{1}{2}e^2$ **(2)**

2009

$x=2$ and $x=-\dfrac{1}{4}$ **(3)**

2010

1. a) $f'(x)=e^x\sin x^2+e^x(2x\cos x^2)$ b) $g'(x)=\dfrac{3x^2(1+\tan x)-x^3\sec^2 x}{(1+\tan x)^2}$

2011

$\dfrac{dy}{dx}=24$ **(4)**

Advanced Higher Topic Past Papers – Integration *Unit 1 Outcome 3*

2002

A6. Use the substitution $x + 2 = 2\tan\theta$ to obtain $\int \frac{1}{x^2 + 4x + 8}\,dx$ **(5)**

2003

A5. Use the substitution $x = 1 + \sin\theta$ to evaluate $\int_0^{\pi/2} \frac{\cos\theta}{(1+\sin\theta)^3}\,d\theta.$ 3 marks

2004

9. Use the substitution $x = (u - 1)^2$ to obtain $\int \frac{1}{\left(1+\sqrt{x}\right)^3}\,dx.$ 5 marks

11. A solid is formed by rotating the curve $y = e^{-2x}$ between $x = 0$ and $x = 1$ through 360° about the x-axis.
Calculate the volume of the solid that is formed. 5 marks

2005

5. Use the substitution $u = 1 + x$ to evaluate $\int_0^3 \frac{x}{\sqrt{1+x}}\,dx.$ 5 marks

2006

6. Find $\int \frac{12x^3 - 6x}{x^4 - x^2 + 1}\,dx.$ 3 marks

2007

10. Use the substitution $u = 1 + x^2$ to obtain $\int_0^1 \frac{x^3}{\left(1+x^2\right)^4}\,dx.$ 5 marks

A solid is formed by rotating the curve $y = \frac{x^{3/2}}{\left(1+x^2\right)^2}$ between $x = 0$ and $x = 1$ through 360° about the x-axis. Write down the volume of this solid. 1 mark

2008

10. A body moves along a straight line with velocity $v = t^3 - 12t^2 + 32t$ and time t.
(a) Obtain the value of its acceleration when $t = 0$. 1 mark

(b) At time $t = 0$, the body is at the origin O. Obtain a formula for the displacement of the body at time t. 2 marks
Show that the body returns to O, and obtain the time, T, when this happens. 2 marks

2009

5. Show that $\int_{\ln\frac{3}{2}}^{\ln 2} \frac{e^x + e^{-x}}{e^x - e^{-x}}dx = \ln\frac{9}{5}$. 4 marks

7. Use the substitution $x = 2\sin\theta$ to obtain the exact value of $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}dx$. 6 marks
(Note that $\cos 2A = 1 - 2\sin^2 A$.)

2010

15. A new board game has been invented and the symmetrical design on the board is made from four identical "petal" shapes. One of these petals is the region enclosed between the curves $y = x^2$ and $y^2 = 8x$ as shown shaded in diagram 1 below.
Calculate the area of the complete design, as shown in diagram 2. 5 marks

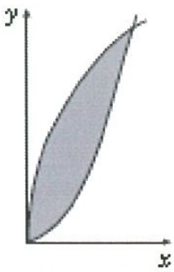


Diagram 1

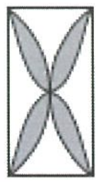


Diagram 2

The counter used in the game is formed by rotating the shaded area shown in diagram 1 above, through 360° about the y-axis. Find the volume of plastic required to make one counter. 1 mark

2011

11. (a) Obtain the exact value of $\int_0^{\pi/4} (\sec x - x)(\sec x + x)dx$ 3 marks

ANSWERS

2002

A6. $\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+c$

2003

$\frac{-1}{8}-\frac{-1}{2}=\frac{3}{8}$

2004

9. $\left(\frac{1}{\left(1+\sqrt{x}\right)^2}-\frac{2}{\left(1+\sqrt{x}\right)}\right)+c$ **(5)**

11. $V=\frac{\pi}{4}\left[1-\frac{1}{e^4}\right]=0.7706$ **(5)**

2005

$2\frac{2}{3}$

2006

$3\ln\left(x^4-x^2+1\right)+c$ **(3)**

2007

$\frac{1}{24}$ **(5)** $\frac{\pi}{24}$ **(1)**

2008

2008

10.a) Acc = 10 **(1)** b) $x(t)=\frac{1}{4}t^4-4t^3+16t^2$ **(2)** c) $t=8$ **(2)**

2009

5. Proof 7. $\frac{\pi}{2}-1$ **(6)**

2010

15. Area $=\frac{32}{3}$ Volume $=\frac{24\pi}{5}=15$

2011

$1-\frac{\pi^3}{192}$

2001

A8. A function f is defined by $f(x)=\dfrac{x^2+6x+12}{x+2}, x\neq -2$.

(a) Express $f(x)$ in the form $ax+b+\dfrac{b}{x+2}$ stating the values of a and b. **(2)**

(b) Write down an equation for each of the two asymptotes. **(2)**

(c) Show that $f(x)$ has two stationary points.
Determine the coordinates and the nature of the stationary points. **(4)**

(d) Sketch the graph of f. **(1)**

(e) State the range of values of k such that the equation $f(x) = k$ has no solution. **(1)**

2002

A8. Express $\dfrac{x^2}{(x+1)^2}$ in the form $A+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}, (x\neq -1)$, stating the values of the constants A, B and C. **(3)**

A curve is defined by $y=\dfrac{x^2}{(x+1)^2}, (x\neq -1)$.

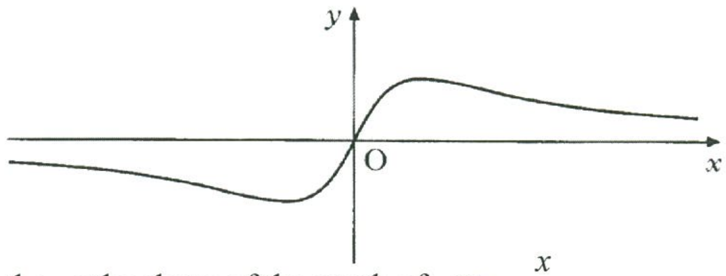
(i) Write down equations for its asymptotes. **(2)**

(ii) Find the stationary point and justify its nature. **(4)**

(iii) Sketch the curve showing clearly the features found in (i) and (ii). **(2)**

2003

A7.



The diagram shows the shape of the graph of $y=\dfrac{x}{1+x^2}$.

Obtain the stationary points of the graph. 4 marks

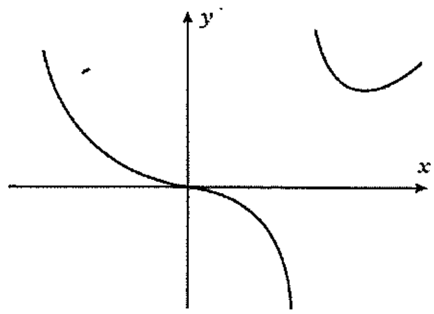
Sketch the graph of $y=\left|\dfrac{x}{1+x^2}\right|$ and identify its three critical points. 3 marks

2004

10. Determine whether the function $f(x)=x^4\sin 2x$ is odd, even or neither.
Justify your answer. 3 marks

2005

11. The diagram shows part of the graph of $y = \frac{x^3}{x-2}, x \neq 2$.



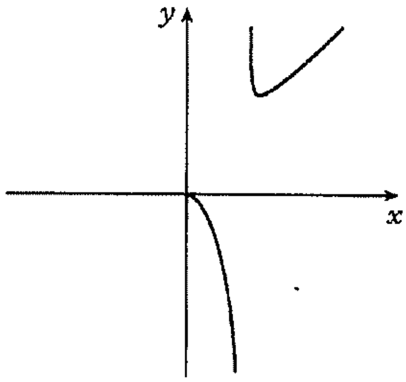
a) Write down the equation of the vertical asymptotes. 1 mark

b) Find the coordinates of the stationary points of the graph of $y = \frac{x^3}{x-2}$. 4 marks

c) Write down the coordinates of the stationary points of the graph of $y = \left|\frac{x^3}{x-2}\right| + 1$. 2 marks

2006

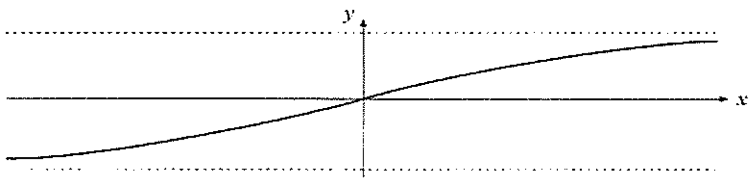
12. The diagram shows part of the graph of a function f which satisfies the following conditions:
    (i) f is an even function;
    (ii) two of the asymptotes of the graph y = f(x) are y = x and x = 1.
    Copy the diagram and complete the graph. Write down equations for the other two asymptotes.



3 marks

2007

16.



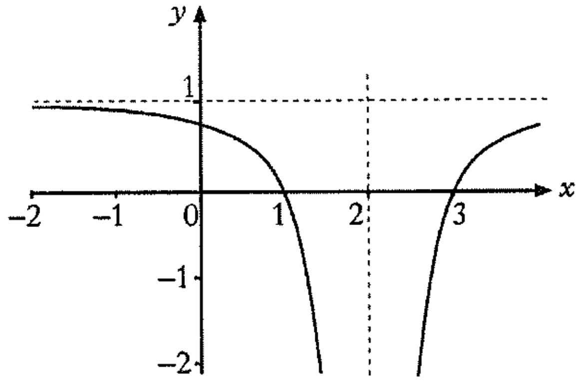
(a) The diagram shows part of the graph of $f(x) = \tan^{-1}2x$ and its asymptotes. State the equations of these asymptotes. 2 marks

(b) Use integration by parts to find the area between $f(x)$, the x-axis and the lines $x = 0, x = \frac{1}{2}$. 5 marks

(c) Sketch the graph of $y = |f(x)|$ and calculate the area between this graph, the x-axis and the lines $x = -\frac{1}{2}, x = \frac{1}{2}$. 3 marks

2008

3. Part of the graph $y = f(x)$ is shown below, where the dotted lines indicate asymptotes. Sketch the graph of $y = -f(x + 1)$ showing its asymptotes. Write down the equations of the asymptotes. 4 marks



2009

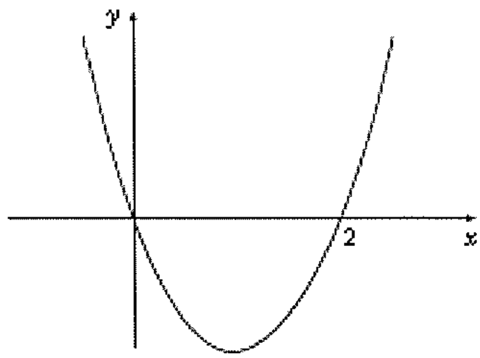
1, 1 marks

13. A function $f(x)$ is defined by $f(x) = \dfrac{x^2 + 2x}{x^2 - 1}$ $(x \neq \pm 1)$.

Obtain equations for the asymptotes of the graph of $f(x)$. 3 marks
Show that $f(x)$ is strictly decreasing function. 3 marks
Find the coordinates of the points where the graph of $f(x)$ crosses
(i) the x-axis and
(ii) the horizontal asymptote. 2 marks
Sketch the graph of $f(x)$, showing clearly all relevant features. 2 marks

2010

10. The diagram below shows part of the graph of a function $f(x)$. State whether $f(x)$ is odd, even or neither. Fully justify your answer.

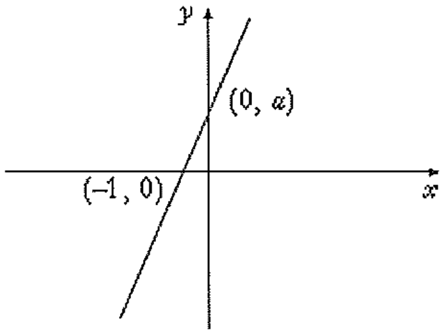


3 marks

2011

Marks

6.



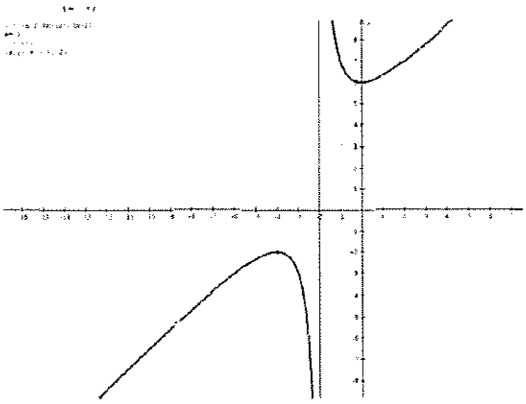
The diagram shows part of the graph of a function $f(x)$. Sketch the graph of $|f^{-1}(x)|$ showing the points of intersection with the axes. **4**

2001

A8. a) $a = 1,\ b = 4$ b) $x = -2,\ \ y = x + 4$

c) Max (-4, -2) Min (0, 6)

d) $-2 < k < 6$

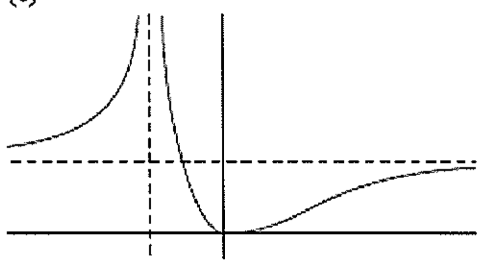


2002

A8. A = 1, B = -2, C = 1 a) $y = 1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}$ $y = 1, x = -1$

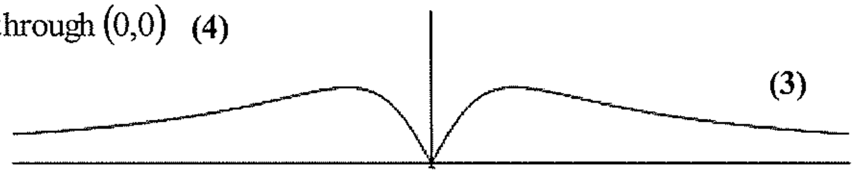
b) Min (0, 0)

(c)



2003

SP $= \left(1, \frac{1}{2}\right)$ and $\left(-1, -\frac{1}{2}\right)$ and passes through $(0,0)$ **(4)**



(3)

2004

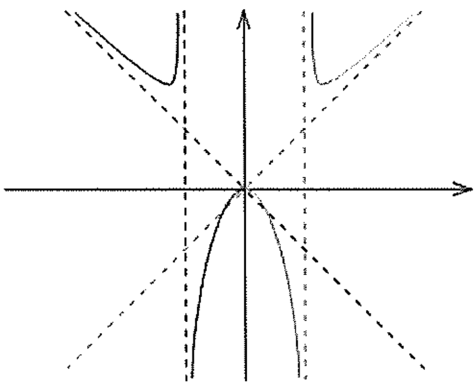
$f(-x) = -f(x)$ therefore is ODD. **(3)**

2005

11. a) $x = 2$ **(1)** b) (0, 0) and (3, 27) **(4)** c) (0, 1) and (3, 28) **(2)**

2006

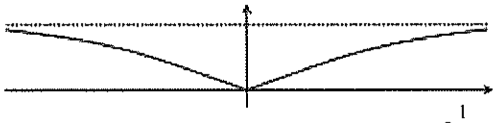
12.



$y = -x;\ x = -1$ **(3)**

2007

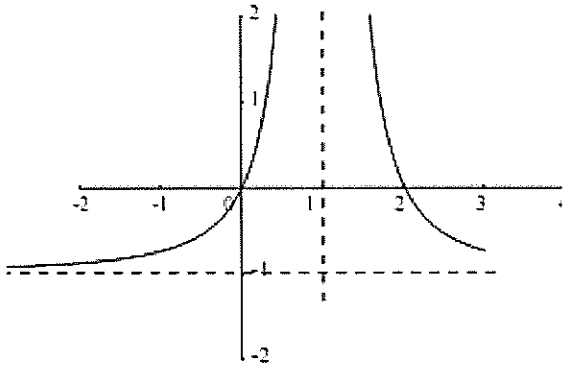
$y = \pm\frac{\pi}{2}$ **(2)** $\frac{\pi}{8} - \frac{1}{4}\ln 2$ **(5)**



$\int_{-\frac{1}{2}}^{\frac{1}{2}} |f(x)|dx = \frac{\pi}{4} - \frac{1}{2}\ln 2$ **(3)**

2008

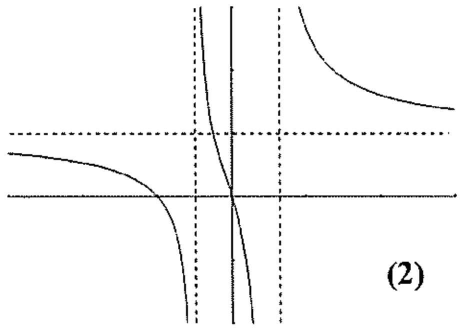
$f'(x) = \frac{-3}{\sqrt{1-9x^2}}$ **(2)** $f'(x) = \frac{3\cos^3\theta}{2\sin\theta}$ **(3)**



2009

$x = -1 \quad x = 1 \quad y = 1$ **(3)** $f'(x) < 0$ therefore always neg grad **(3)**

$x = 0 \quad x = -2$ Horizontal Asy $x = -\frac{1}{2}$ **(2)**



(2)

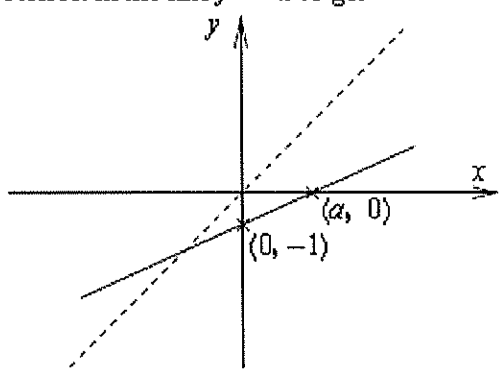
2010

10. The graph is not symmetrical about y – axis therefore it is NOT an even function.
The graph does not have half turn rotational symmetry therefore it is NOT an odd function.
The function is neither odd nor even.

2011

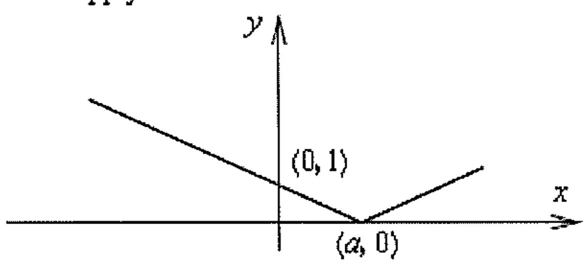
6. **(4)**

Reflect in the line $y = x$ to get



1 for position
1 for coordinates

Now apply the modulus function



1 for shape
1 for coordinates

2001

A1. Use Gaussian elimination to solve the following system of equations. **(5)**

$$\begin{aligned} x+y+z&=10\\ 2x-y+3z&=4\\ x+2z&=20 \end{aligned}$$

2002

A1. Use Gaussian elimination to solve the following system of equations. **(5)**

$$\begin{aligned} x+y+3z&=2\\ 2x+y+z&=4\\ 3x+2y+5z&=5 \end{aligned}$$

2003

A6. Use elementary row operations to reduce the following system of equations to upper triangular form

$$\begin{aligned} x+y+3z&=1\\ 3x+ay+z&=1\\ x+y+z&=-1 \end{aligned}$$

2 marks

2 marks

Hence express x, y and z in terms of the parameter a.

Explain what happens when a = 3. 2 marks

2005

6. Use Gaussian elimination to solve the system of equations below when $\lambda \neq 2$:

$$\begin{aligned} x+y+2z&=1\\ 2x+\lambda y+z&=0\\ 3x+3y+9z&=5 \end{aligned}$$

4 marks

2 marks

Explain what happens when $\lambda = 2$.

2006

9. Use Gaussian elimination to obtain solutions of the equations

$$\begin{aligned} 2x-y+2z&=1\\ x+y-2z&=2\,.\\ x-2y+4z&=-1 \end{aligned}$$

5 marks

2009

16. (a) Use Gaussian elimination to solve the following system of equations 5 marks

$$\begin{aligned} x+y-z&=6\\ 2x-3y+2z&=2\\ -5x+2y-4z&=1. \end{aligned}$$

2010

Use Gaussian elimination to show that the set of equations

$$\begin{aligned} x-y+z&=1\\ x+y+2z&=0\\ 2x-y+az&=2 \end{aligned}$$

has a unique solution when $a \neq 2\cdot5$.

5 marks

Explain what happens when $a = 2.5$.

1 mark

2001

A1. $x = -26$, $y = 13$, $z = 23$

2002

A1. $x = 2$, $y = -3$, $z = 1$.

2003

A6 When $a = 3$, we get $z = \frac{1}{4}$ from the second equation but $z = 1$ from the third, ie inconsistent.

2005

$x = -\frac{1}{3}$; $y = 0$; $z = \frac{2}{3}$. 2nd and 3rd rows of second matrix are same - infinite number of solutions.

2006

$x = 1$; $y = 1 + 2t$; $z = t$

2009

$x = 3$, $y = -2$, $z = -5$

2010

$x = \frac{a-4}{2a-5}$, $y = \frac{2-a}{2a-5}$, $z = \frac{1}{2a-5}$ When $a = 2.5$ there are no solutions.