Recurrence Relations Objective 1

1. A sequence is defined by the recurrence relation $u_{n+1} = -3u_n + 7$ with $u_0 = 2$.

   What is the value of u_2?

   A. -1

   B. 1

   C. 4

   D. 10 **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.4	C	0.85	0.38	NC	A11	HSN 157

$u_1 = -3 \times 2 + 7 = -6 + 7 = 1$

$u_2 = -3 \times 1 + 7 = -3 + 7 = 4.$ Option $\boxed{C}$

2. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{4}u_n + 8$ with $u_0 = 32$.

   Evaluate u_2.

   A. 10

   B. 12

   C. 16

   D. 32 **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	1.4	C	0.83	0.35	NC	A11	HSN 137

$u_1 = \frac{1}{4}u_0 + 8 = \frac{1}{4} \times 32 + 8 = 8 + 8 = 16.$

$u_2 = \frac{1}{4}u_1 + 8 = \frac{1}{4} \times 16 + 8 = 4 + 8 = 12.$ Option $\boxed{B}$

3. A sequence is defined by the recurrence relation $u_{n+1} = au_n + b$, where a and b are constants.

   Given that $u_0 = 4$ and $u_1 = 8$, find an expression for a in terms of b.

   A. $a = \frac{1}{2} - \frac{1}{8}b$

   B. $a = 2 - \frac{1}{4}b$

   C. $a = \frac{1}{2} + \frac{1}{8}b$

   D. $a = 2 + \frac{1}{4}b$ **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	1.4	C	0.62	0.5	CN	A10, A14	HSN 060

$u_0 = 4$

$u_1 = a \times 4 + b = 4a + b = 8$

So $4a = 8 - b$

$a = \frac{8}{4} - \frac{b}{4}$

$= 2 - \frac{1}{4}b$ Option $\boxed{B}$

4. Two sequences are defined by

$$u_{n+1} = \tfrac{1}{2}u_n + 7 \text{ and}$$
$$v_{n+1} = -v_n + 2,$$

with $u_0 = -4$ and $v_0 = 10$.

Here are two statements about the sequences:

I. u_n tends to a limit as $n \to \infty$.

II. v_n tends to a limit as $n \to \infty$.

Which of the following is true?

A. neither statement is correct

B. only statement I is correct

C. only statement II is correct

D. both statements are correct **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	1.4	C	0.45	0.55	CN	A12	HSN 166

Since $-1 < \frac{1}{2} < 1$, u_n tends to a limit as $n \to \infty$.

$v_0 = 10$, $v_1 = -8$, $v_2 = 10$, ... so v_n alternates between two numbers – there is no limit.

Note:
Just because $-1 < a < 1$ is not satisfied, we cannot conclude that there is no limit.
e.g. $u_{n+1} = -u_n + 2$
$u_0 = 1$.

Option $\boxed{B}$

5. A sequence is defined by the recurrence relation $u_{n+1} = \frac{2}{5}u_n + 6$ with $u_0 = -10$.

   What is the limit of the sequence?

   A. 10

   B. $\frac{2}{5}$

   C. $-\frac{2}{25}$

   D. -30 **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.4	C	0.94	0.14	NC	A13	HSN 088

A limit exists since $-1 < \frac{2}{5} < 1$.

Method 1 $l = \frac{b}{1-a}$ where $a = \frac{2}{5}$, $b = 6$

$$= \frac{6}{1-\frac{2}{5}}$$

$$= \frac{6}{3/5}$$

$$= 10.$$

Method 2 As $n \to \infty$, $u_{n+1} = u_n = l$.

$$l = \frac{2}{5}l + 6$$

$$\frac{3}{5}l = 6$$

$$l = 10.$$

Option $\boxed{A}$

6. A sequence is defined by the recurrence relation $u_{n+1} = au_n + \frac{3}{2}$, with $u_0 = 5$.

   Given that this sequence has limit 1, what is the value of a?

   A. $-\frac{1}{2}$

   B. $-\frac{1}{3}$

   C. $\frac{1}{3}$

   D. $\frac{1}{2}$ **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.4	C	0.48	0.43	NC	A13, A14	HSN 141

Method 1 $l = \dfrac{b}{1-a} = \dfrac{3/2}{1-a} = 1.$

So $1 - a = \dfrac{3}{2}$

$a = 1 - \dfrac{3}{2}$

$= -\dfrac{1}{2}.$

Method 2 As $n \to \infty$, $u_{n+1} \to u_n \to 1.$

So $1 = a \times 1 + \dfrac{3}{2}$

$a = 1 - \dfrac{3}{2}$

$= -\dfrac{1}{2}.$

Option $\boxed{A}$

[END OF QUESTIONS]