

Marked Homework 11 - Sequences

1. (a) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.
Write down the values of u_1 and u_2 . 1
- (b) A second sequence is given by $4, 5, 7, 11, \dots$
It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.
Find the values of p and q . 3
- (c) Either the sequence in (a) or the sequence in (b) has a limit.
(i) Calculate this limit.
(ii) Why does the other sequence not have a limit? 3

- [SQA] 2. Two sequences are defined by these recurrence relations:
 $u_{n+1} = 3u_n - 0.4$ with $u_0 = 1$, $v_{n+1} = 0.3v_n + 4$ with $v_0 = 1$.
- (a) Explain why only one of these sequences approaches a limit as $n \rightarrow \infty$. 1
- (b) Find algebraically the exact value of the limit. 2
- (c) For the other sequence, find
(i) the smallest value of n for which the n^{th} term exceeds 1000, and
(ii) the value of that term. 2

- [SQA] 3. A sequence is defined by the recurrence relation $u_{n+1} = 0.3u_n + 5$ with first term u_1 .
- (a) Explain why this sequence has a limit as n tends to infinity. 1
- (b) Find the **exact** value of this limit. 2

- [SQA] 4. Trees are sprayed weekly with the pesticide, KILLPEST, whose manufacturers claim it will destroy 65% of all pests. Between the weekly sprayings it is estimated that 500 new pests invade the trees.
- A new pesticide, PESTKILL, comes onto the market. The manufacturers claim that it will destroy 85% of existing pests but it is estimated that 650 new pests per week will invade the trees.
- Which pesticide will be more effective in the long term? 7

- [SQA] 5. Secret Agent 004 has been captured and his captors are giving him a 25 milligram dose of a truth serum every 4 hours. 15% of the truth serum present in his body is lost every hour.
- (a) Calculate how many milligrams of serum remain in his body after 4 hours (that is immediately before the second dose is given). (3)
- (b) It is known that the level of serum in the body has to be continuously above 20 milligrams before the victim starts to confess. Find how many doses are needed before the captors should begin their interrogation. (3)
- (c) Let u_n be the amount of serum (in milligrams) in his body just after his n^{th} dose. Show that $u_{n+1} = 0.522u_n + 25$. (1)
- (d) It is also known that 55 milligrams of this serum in the body will prove fatal, and the captors wish to keep Agent 004 alive. Is there any maximum length of time for which they can continue to administer this serum and still keep him alive? (4)

- [SQA] 6. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2v_n + 16$.

The two sequences approach the same limit as $n \rightarrow \infty$.

Determine the value of a and evaluate the limit.

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- [SQA] 7. Two sequences are defined by the recurrence relations

$$\begin{aligned} u_{n+1} &= 0.2u_n + p, & u_0 &= 1 & \text{and} \\ v_{n+1} &= 0.6v_n + q, & v_0 &= 1. \end{aligned}$$

If both sequences have the same limit, express p in terms of q .

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[END OF QUESTIONS]