## Marked Homework 10 - Circles 2

[SQA]
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2.

A spherical hot-air balloon has radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.

Co-ordinate axes are chosen as shown in the diagram. One of the cables is represented by PB and PBA is a straight line.
(a) Find the equation of the cable PB.
(b) State the equation of the circle representing the balloon.
(c) Prove that this cable is a tangent to the balloon and find the co-ordinates of the point $P$.

3. Circle $P$ has equation $x^{2}+y^{2}-8 x-10 y+9=0$. Circle $Q$ has centre $(-2,-1)$ and radius $2 \sqrt{2}$.
(a) (i) Show that the radius of circle P is $4 \sqrt{2}$.
(ii) Hence show that circles P and Q touch.
(b) Find the equation of the tangent to the circle $Q$ at the point $(-4,1)$.
(c) The tangent in (b) intersects circle P in two points. Find the $x$-coordinates of the points of intersection, expressing you answers in the form $a \pm b \sqrt{3}$.
4. (a) (i) Show that the line with equation $y=3-x$ is a tangent to the circle with equation $x^{2}+y^{2}+14 x+4 y-19=0$.
(ii) Find the coordinates of the points of contact, P.
(b) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre C.


The line $y=3-x$ is a common tangent at the point P .
The radius of the larger circle is three times the radius of the smaller circle.
Find the equation of the smaller circle.
5. Find the possible values of $k$ for which the line $x-y=k$ is a tangent to the circle $x^{2}+y^{2}=18$.
6.
(a) Show that the point $P(5,10)$ lies on circle $C_{1}$ with equation $(x+1)^{2}+(y-2)^{2}=100$.
(b) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q .

(c) Two circles, $C_{2}$ and $C_{3}$, touch circle $C_{1}$ at $Q$.

The radius of each of these circles is twice the radius of circle $C_{1}$.
Find the equations of circles $C_{2}$ and $C_{3}$.

