Higher Mathematics - Practice Examination G

Please note … the format of this practice examination is the same as the current format. The paper timings are the same, however, there are some differences in the marks allocated. Calculators may only be used in Paper 2.

MATHEMATICS

Higher Grade - Paper I (Non~calculator)

Time allowed - 1 hour 10 minutes

Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |a|\,|b|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or

$\boldsymbol{a} \cdot \boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$
\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

Table of standard derivatives:

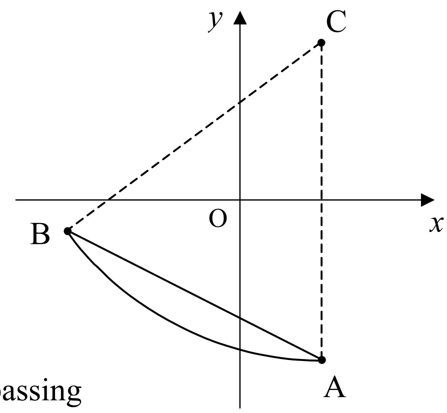
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. The diagram shows an arc AB of the circle with its centre at C. The coordinates of A and B are $(4, -7)$ and $(-8, -1)$ respectively.



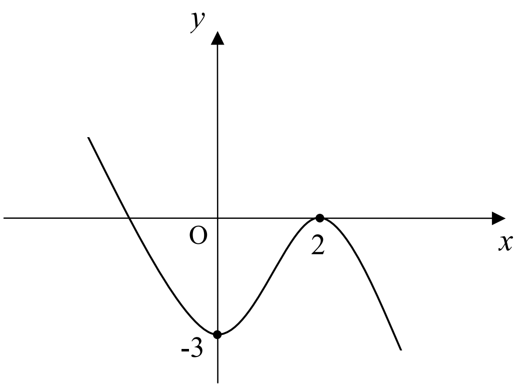
   (a) Find the equation of the perpendicular bisector of the chord AB. **4**

   (b) Hence establish the coordinates of C, given that C is vertically above A. **2**

   (c) Write down the equation of the circle, centre C, passing through the points A and B. **2**

2. For what value(s) of p does the equation $(4p+1)x^2 - 3px + 1 = 0$ have equal roots? **4**

3. The diagram below shows part of the the graph of $y = g(x)$.
   The function has stationary points at $(0, -3)$ and $(2, 0)$ as shown.



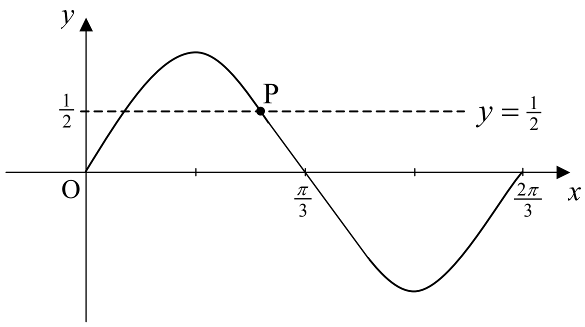
   Sketch the graph of the related function $y = g(-x) + 3$. **3**

4. A function is defined as $f(\theta) = \cos 2\theta + 3\sin^2\theta$ for $0 < \theta < \frac{\pi}{2}$.

   (a) Show that $f'(\theta) = \sin 2\theta$. **4**

   (b) Hence calculate the rate of change of the function at $\theta = \frac{\pi}{12}$. **1**

5. The daigram shows the graph of $y = \sin 3x$, for $0 \le x \le \frac{2\pi}{3}$, and the line with equation $y = \frac{1}{2}$.



Establish the coordinates of the point P. **3**

6. A curve has as its derivative $\frac{dy}{dx} = 8x - 3$.

Given that the point $(1, -3)$ lies on this curve, express y in terms of x. **4**

7. A sequence of numbers is defined by the recurrence relation $U_{n+1} = kU_n + c$, where k and c are constants.

(a) Given that $U_2 = 70$, $U_3 = 65$ and $U_4 = 62 \cdot 5$, find **algebraically**, the values of k and c. **3**

(b) Hence find the limit of this sequence. **2**

(c) Express the difference between the fifth term and the limit of this sequence as a percentage of the limit, correct to the nearest percent. **2**

8. A function is given as $f(x) = 3x^3 - 9x^2 + 27x$ and is defined on the set of real numbers.

(a) Show that the derivative of this function can be expressed in the form $f'(x) = a\left[(x-b)^2 + c\right]$ and write down the values of *a*, *b* and *c*. **4**

(b) Explain why this function has no stationary points and is in fact increasing for **all** values of x. **2**

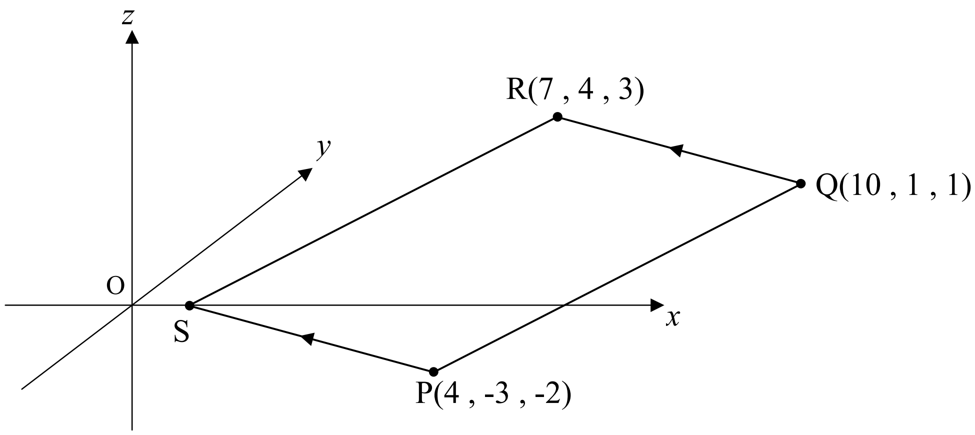
9. The functions $f(x) = x^2 - 9$ and $h(x) = 3 + 2x$ are defined on the set of real numbers.

(a) Evaluate $h\big(f(3)\big)$. **1**

(b) Find an expression , in its simplest form, for $f\big(h(x)\big)$. **2**

(c) For what value(s) of x does $f\big(h(x)\big) = f(x)$? **2**

10. Three vertices of the quadrilateral PQRS are P(4 , -3 , -2) Q(10 , 1 , 1) and R(7 , 4 , 3).

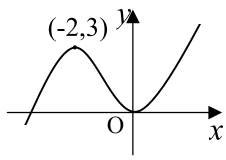


(a) Given that $\overrightarrow{QR} = \overrightarrow{PS}$, establish the coordinates of S. **2**

(b) Hence show that angle PSR is a right angle. **3**

11. Given that $\log_2(5x - 1) - \log_2 x = 2$, find the value of x . **3**

[END OF QUESTION PAPER]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**(a) ans:** $y = 2x$ **4 marks** •1 For mid-point of AB •2 For gradient of AB •3 For gradient of perpendicular •4 For equation of bisector **(b) ans:** C(4,8) **2 marks** •1 For knowing to use x_A •2 For sub. in equation to answer **(c) ans:** $(x-4)^2+(y-8)^2 = 225$ **2 marks** •1 For finding radius •2 For sub. in standard equ. to answer	(a) •1 M(-2,-4) •2 $m_{AB} = -\frac{1}{2}$ •3 $m_1 \times m_2 = -1$, $m_{bis.} = 2$ •4 $y+4 = 2(x+2) \Rightarrow y = 2x$ (b) •1 $x_C = x_A = 4$ •2 $y = 2(4) = 8 \;\therefore\; C(4,8)$ (c) •1 CA is vertical = radius = 15 units •2 C(4,8) , $r = 15$ in equ……. $(x-a)^2+(y-b)^2 = r^2$
2.	**ans:** $p = -\frac{2}{9}$, $p = 2$ **4 marks** •1 For discr. = 0 (stated or implied) •2 For selecting a, b and c •3 Substituting and simplifying •4 Factorising to answers	•1 For equal roots $b^2-4ac = 0$ •2 $a = 4p+1$, $b = -3p$, $c = 1$ •3 $(-3p)^2-(4(4p+1).1) = 0$ $\Rightarrow 9p^2-16p-4 = 0$ •4 $(9p+2)(p-2) = 0 \Rightarrow p = -\frac{2}{9}$ *or* 2
3.	**ans:** diagram **3 marks** •1 For reflection in y-axis •2 For translating …. 3 up •3 For annotating final sketch	
4.	**(a) ans:** proof **4 marks** •1 For differentiating first term •2 For differentiating second term •3 common factor (isolating double angle) •4 for double angle + simplifying **(b) ans:** $\frac{1}{2}$ **1 marks** •1 answer	(a) •1 $-2\sin 2\theta$ .... •2 …….. + $6\sin\theta\cos\theta$ •3 ……...+ $3(2\sin\theta\cos\theta)$ •4 $-2\sin\theta+3(\sin 2\theta) = \sin 2\theta$ (b) •1 $f'(\frac{\pi}{12}) = \sin(2\times\frac{\pi}{12}) = \sin\frac{\pi}{6} = \frac{1}{2}$
5.	**ans:** $P(\frac{5\pi}{18}, \frac{1}{2})$ **3 marks** •1 For equating •2 Angles for sin3x (choosing quad.) •3 Answer	•1 $\sin 3x = \frac{1}{2}$ •2 quad ½ $\therefore\; 3x = \frac{\pi}{6}$ *or* $\frac{5\pi}{6}$ •3 $x = \frac{5\pi}{18}$ ……. $P(\frac{5\pi}{18}, \frac{1}{2})$
6.	**ans:** $y = 4x^2-3x-4$ **4 marks** •1 For setting up integral •2 For integrating •3 For substituting •4 Correct answer	•1 $y = \int 8x-3\; dx$ •2 $y = \frac{8x^2}{2}-3x+c \Rightarrow 4x^2-3x+c$ •3 $-3 = 4(1^2)-3(1)+c$ •4 $c = -4$ to answer

	Give 1 mark for each •	Illustration(s) for awarding each mark
7.	**(a) ans:** $k=\frac{1}{2}$, $c=30$ **3 marks** •1 Setting up a system of equ. •2 Finding k •3 Finding c **(b) ans:** 60 **2 marks** •1 Knowing how to find limit •2 Calculating limit **(c) ans:** 2% **2 marks** •1 For calculating U_5 •2 For percentage calculation	(a) •1 $U_3=kU_2+c \Rightarrow 65=70k+c$ $U_4=kU_3+c \Rightarrow 62\cdot5=65k+c$ •2 $5k=2\cdot5 \Rightarrow k=\frac{1}{2}$ •3 $c=30$ (b) •1 $L=\dfrac{b}{1-a}$, or equivalent •2 $L=30/(1-\frac{1}{2})=60$ (c) •1 $U_5=\frac{1}{2}(62\cdot5)+30=61\cdot25$ •2 $\frac{1\cdot25}{60}\times100 \approx 2\%$
8.	**(a) ans:** $a=9$, $b=1$, $c=2$ **4 marks** •1 For differentiating •2 For common factor •3 For the square $(x-1)^2$ •4 For $c=2$ (no need to list a,b and c) **(b) ans:** explanation **2 marks** •1 For statement on solving to zero •2 Derivative always +ve, always increasing	(a) •1 $f'(x)=9x^2-18x+27$ •2 $9\left[x^2-2x+3\right]$ •3 $9\left[(x-1)^2\ ....\right]$ •4 $9\left[..........\ -1+3\right]=9\left[(x-1)^2+2\right]$ (no marks off if $b=-1$) (b) •1 $(x-1)^2+2=0$ has no solution •2 $(x-1)^2+2$ always +ve , always incr.
9.	**(a) ans:** 3 **1 mark** •1 answer **(b) ans:** $f(h(x))=12x+4x^2$ **2 marks** •1 For substitition •2 Simplifying to answer **(c) ans:** $x=-3$, $x=-1$ **2 marks** •1 For equating •2 For solving to answers	(a) •1 $f(3)=0$, $h(0)=3$ (or equiv.) (b) •1 $f(h(x))=(3+2x)^2-9$ •2 $f(h(x))=12x+4x^2$ (c) •1 $12x+4x^2=x^2-9$ •2 $3x^2+12x+9=0$ $3(x+3)(x+1)=0$ … x = -3 or -1
10.	**(a) ans:** S(1,0,0) **2 marks** •1 For finding displacement $\vec{QR}$ •2 For establishing coordinates of S **(b) ans:** proof **3 marks** •1 For knowing $\vec{SR}.\vec{SP}=0$, for R.A. (stated or implied) •2 For both displacements •3 For scalar product calculation to zero	(a) •1 $\vec{QR}=r-a=\begin{pmatrix}7\\4\\3\end{pmatrix}-\begin{pmatrix}10\\1\\1\end{pmatrix}=\begin{pmatrix}-3\\3\\2\end{pmatrix}$ •2 $s=\begin{pmatrix}4\\-3\\-2\end{pmatrix}+\begin{pmatrix}-3\\3\\2\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, or equiv. (b) •1 For right-angle $\vec{SR}.\vec{SP}=0$ •2 $\vec{SR}.\vec{SP}=\begin{pmatrix}6\\4\\3\end{pmatrix}\begin{pmatrix}3\\-3\\-2\end{pmatrix}=...........$ •3 $=18-12-6=0$, $\therefore r-angled$
11.	**ans:** $x=1$ **3 marks** •1 For combining logs •2 For removing logs •3 For finding x	•1 $\log_2\left(\dfrac{5x-1}{x}\right)=2$ •2 $2^2=\dfrac{5x-1}{x}$ (or equivalent) •3 $4x=5x-1 \Rightarrow x=1$

Total 53 marks

Higher Mathematics - Practice Examination G

Please note … the format of this practice examination is the same as the current format. The paper timings are the same, however, there are some differences in the marks allocated. Calculators may only be used in this paper.

MATHEMATICS

Higher Grade - Paper II

Time allowed - 1 hour 30 minutes

Read Carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\,, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a\,, b)$ and radius r.

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |a|\,|b|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or

$\boldsymbol{a} \cdot \boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

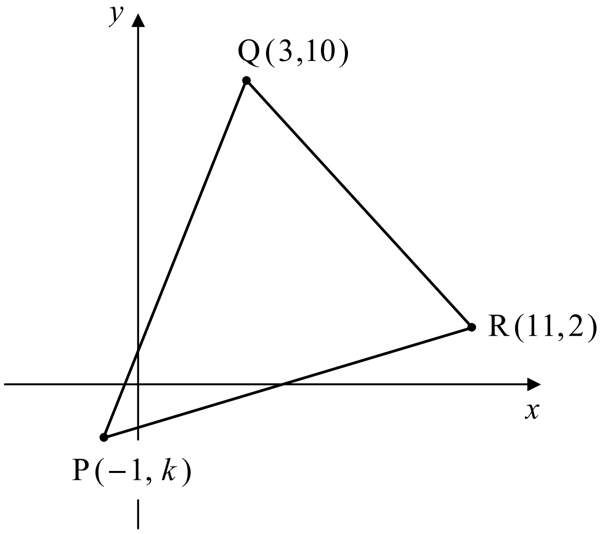
$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. Triangle PQR has vertices $P(-1, k)$, $Q(3,10)$ and $R(11,2)$ as shown.



(a) Given that the gradient of side PQ is 3, find the equation of PQ. **2**

(b) Hence find *k*, the *y*-coordinate of vertex P. **1**

(c) Find the equation of the median from P to QR. **3**

(d) Show that this median is at right-angles to side QR.
What type of triangle is PQR? **3**

2. Evaluate $f'(4)$ when $f(x) = \dfrac{x - 2\sqrt{x}}{x^2}$. **5**

3. Two functions are defined as $f(x) = ax^2 - 2b$ and $h(x) = \dfrac{2x - 6b}{3}$,
where a is a constant .

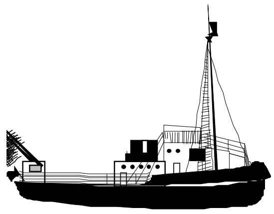
(a) Given that $f(2) = h(2)$, show clearly that $a = \frac{1}{3}$. **3**

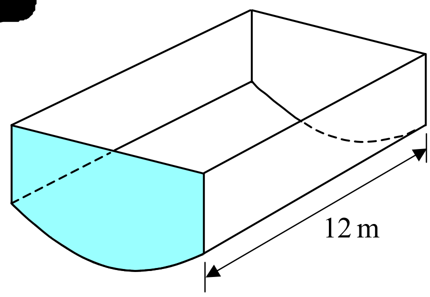
(b) If $b = px - 6$, show that $f(x) = \frac{1}{3}x^2 - 2px + 12$. **1**

(c) Hence state the values of p for which $f(x) = 0$ has no real roots. **4**

4. A fishing boat's fish hold is in the shape of the prism shown opposite.

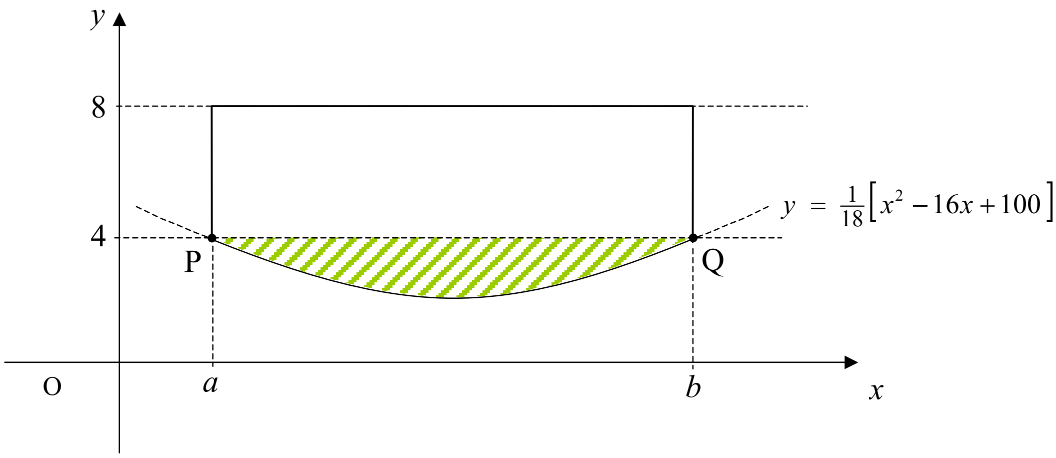
The length of the hold is 12 metres.





The cross-section of the hold is represented in the coordinate diagram below.

All the units are in metres, with the floor of the hold represented by the curve $y = \frac{1}{18}\left[x^2 - 16x + 100\right]$.



(a) Find the values of a and b, the x-coordinates of P and Q. **3**

(b) Show clearly that the area between the line PQ and the curve $y = \frac{1}{18}\left[x^2 - 16x + 100\right]$ can be calculated by evaluating the integral: $A = \frac{1}{18}\int_a^b (16x - x^2 - 28)\,dx$. **2**

(c) Calculate this area in square metres. **4**

(d) Hence calculate the **volume** of the hold, in cubic metres, by first establishing the **total** cross-sectional area of the hold. **2**

5. Two vectors are defined as $V_1 = \sqrt{2}\boldsymbol{i} + 3\boldsymbol{j} - \sqrt{5}\boldsymbol{k}$ and $V_2 = \sqrt{3}\boldsymbol{i} + \sqrt{6}\boldsymbol{j}$.

Calculate the angle between these two vectors to the nearest degree. **5**

6. Certain radioisotopes are used as *tracers*, to track down diseased tissue within the body, and then be absorbed, to act as a long-term radio-therapy treatment. Their passage through the body and mass is ascertained by means of a Geiger-Müller counter.

During trials of a particular radioisotope the following information was obtained.

- *the isotope loses* 3% *of its mass every hour*
- *the maximum recommended mass in the bloodstream is* 165mgs
- 100mgs *is the smallest mass detectable by the Geiger-Müller counter*

(a) An intial dose of 150mgs of the isotope is injected into a patient.

Would the mass remaining after 12 hours still be detectable by the Geiger-Müller counter? **3**

Your answer must be accompanied by appropriate working.

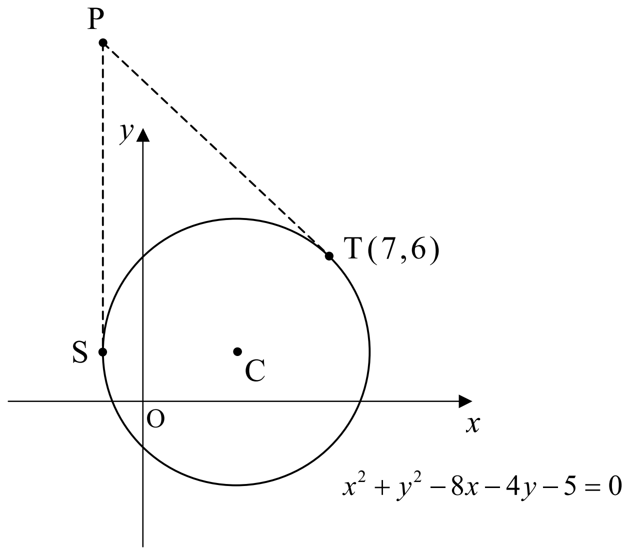
(b) **After the initial dose**, top-up injections of 50mgs are given every 12 hours.

Comment on the long-term suitabilty of this plan. **4**

Your answer must be accompanied by appropriate working.

7. The diagram shows a circle, centre C, with equation $x^2 + y^2 - 8x - 4y - 5 = 0$.

Two common tangents have been drawn from the point P to the points S and T$(7,6)$ on the circle.



(a) Find the centre and radius of the circle. **2**

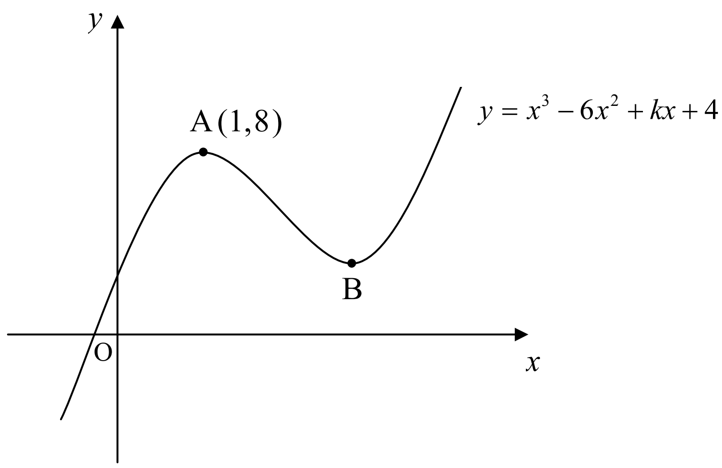
(b) Hence find the equation of the tangent PT. **3**

(c) Given now that the tangent PS is parallel to the *y*-axis, determine the coordinates of S and P. **3**

8. Solve algebraically the equation

$$3\sin x^o = 1 - \sqrt{7}\cos x^o \ , \quad \text{where} \ \ 0 \le x < 180 \, .$$ **7**

9. The curve below has as its equation $y = x^3 - 6x^2 + kx + 4$, where k is a constant.
$\text{A}(1,8)$ is a stationary point.



(a) Using the x-coordinate of A, to help you, find the value of k. **4**

(b) Hence find the coordinates of the other stationary point at B. **3**

[END OF QUESTION PAPER]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**(a) ans:** $y = 3x + 1$ **2 marks** •1 For using Q •2 For answer	(a) •1 Q(3,10) , $m = 3$ •2 $y - 10 = 3(x - 3)$
	(b) ans: $k = -2$ **1 mark** •1 For subst. to answer	(b) •1 $y = 3(-1) + 1 = -2 = k$
	(c) ans: $y = x - 1$ **3 marks** •1 For mid-point of QR •2 For calculating gradient •3 answer	(c) •1 $M_{QR}(7,6)$ •2 $m_{med} = \frac{6-(-2)}{7-(-1)} = 1$ •3 $y - 6 = 1(x - 7)$
	(d) ans: proof , isosceles **3 marks** •1 For knowing $m_1.m_2 = -1$ •2 For calculation to prove •3 For isosceles (no explanation required)	(d) •1 If perp… $m_{QR} \times m_{Pm} = -1$ (*stated or implied*) •2 $-1 \times 1 = -1$ •3 isosceles
2.	**ans:** $f'(4) = \frac{1}{32}$ **5 marks** •1 For preparing to differentiate •2 Differentiating first term •3 Differentiating second term •4 Subst. x = 4 in derivative •5 Calculating answer	•1 $f(x) = x^{-2}(x - 2x^{\frac{1}{2}})$ $= x^{-1} - 2x^{-\frac{3}{2}}$ •2 $f'(x) = -x^{-2}$ .... •3 $f'(x) =$ .... $3x^{-\frac{5}{2}}$ •4 $f'(x) = -\frac{1}{4^2} + \frac{3}{4^{\frac{5}{2}}}$ •5 $f'(4) = -\frac{2}{32} + \frac{3}{32} = \frac{1}{32}$
3.	**(a) ans:** proof **3 marks** •1 For sub 2 in *f* and *h* •2 For equating •3 For solving to required answer	(a) •1 $f(2) = 4a - 2b$, $h(2) = \frac{4-6b}{3}$ •2 $4a - 2b = \frac{4-6b}{3}$ •3 $3(4a - 2b) = 4 - 6b$ $12a - 6b = 4 - 6b$ ....$a = \frac{1}{3}$
	(b) ans: proof **1 mark** •1 For sub. for *a* and *b* and adjusting to required answer	(b) •1 $f(x) = \frac{1}{3}x^2 - 2(px - 6)$ $f(x) = \frac{1}{3}x^2 - 2px + 12$
	(c) ans: $-2 < p < 2$ **4 marks** •1 For discr. statement (or implied) •2 For values of *a*, *b* and *c* •3 For subst. and factorising •4 For final statement (worded ans. o.k.)	(c) •1 for no real roots $b^2 - 4ac < 0$ •2 $a = \frac{1}{3}$, $b = -2p$, $c = 12$ •3 $(-2p)^2 - (4.\frac{1}{3}.12) < 0$ $4(p - 2)(p + 2) < 0$ •4 p has to lie between -2 and 2

	Give 1 mark for each •	Illustration(s) for awarding each mark
4.	**(a) ans:** $a = 2$, $b = 14$ **3 marks** •1 For sub. 4 for y in order to solve •2 manipulating equation to zero •3 factorising and answers	(a) •1 $4 = \frac{1}{18}(x^2 - 16x + 100)$ •2 $x^2 - 16x + 28 = 0$ •3 $(x-2)(x-14) = 0 \Rightarrow x = 2$, $x = 14$
	(b) ans: proof **2 marks** •1 Strategy of line minus curve •2 Constant out + tidy to required answer	(b) •1 $A = \int 4 - \frac{1}{18}(x^2 - 16x - 100)\, dx$ •2 $A = \frac{1}{18}\int 72 - (x^2 - 16x + 100)\, dx$ $A = \frac{1}{18}\int_2^{14} (16x - x^2 - 28)\, dx$
	(c) ans: 16 m^2 **4 marks** •1 For integrating (all 3 terms) •2 For substituting •3 For simplifying each part •4 For calculating answer	(c) •1 $A = \frac{1}{18}\left[8x^2 - \frac{x^3}{3} - 28x\right]_2^{14}$ •2 .... $A = \frac{1}{18}\left[(8(14^2) - \frac{14^3}{3} - 28(14)) - (8(2^2) - \frac{2^3}{3} - 28(2))\right]$ •3 $A = \frac{1}{18}[1200 - 912]$ •4 $A = \frac{1}{18}[288] = 16$
	(d) ans: 768 m^3 **2 marks** •1 For total surface area •2 For volume	(d) •1 $A = 16 + (4 \times 12) = 64\ m^2$ •2 $V = 64 \times 12 = 768\ m^3$
5.	**ans:** 35^o **5 marks** •1 consruct appropriate vectors •2 strategy of $\cos\theta = ...$ •3 calculate scalar product •4 process denominator (magnitudes) •5 calculate angle (*rounding only a guide*)	•1 $V_1 = \begin{pmatrix} \sqrt{2} \\ 3 \\ -\sqrt{5} \end{pmatrix}$, $V_2 = \begin{pmatrix} \sqrt{3} \\ \sqrt{6} \\ 0 \end{pmatrix}$ •2 $\cos\theta = ...$ (formula may only appear when numbers are subst.) •3 $V_1.V_2 = \sqrt{6} + 3\sqrt{6} + 0 = 4\sqrt{6}$ •4 $\lvert V_1 \rvert \times \lvert V_2 \rvert = \sqrt{16} \times \sqrt{9} = 12$ •5 $\cos\theta = \frac{4\sqrt{6}}{12} \quad \therefore \quad \theta = 35 \cdot 3^o \approx 35^o$
6.	**(a) ans:** Yes , $104 \cdot 08 > 100$ **3 marks** •1 For taking $a = 0 \cdot 97$ •2 For calculation •3 For consistent answer	(a) •1 $a = 0 \cdot 97$ •2 $U_{12} = (0 \cdot 97)^{12} \times 150 = 104 \cdot 08$ •3 Yes , since $U_{12} > 100$
	(b) ans: Plan o.k., over the long-term between $113 \cdot 3$ *and* $163 \cdot 3$ mgs **4 marks** •1 For attempting lines of working •2 For finding the limit •3 For being aware of the lower limit as well as the upper limit •4 Consistent comment on findings	(b) •1 $U_1 \rightarrow U_5$ below .... 154·08, 156·91, 158·87, 160·23, 161·17 •2 $L = \frac{50}{1 - (0 \cdot 97)^{12}} = 163 \cdot 31$ •3 lower limit = $163 \cdot 31 - 50 = 113 \cdot 31$ •4 Over the long-term the amount present would always be between 113·31 and 163·31 which is ideal.

	Give 1 mark for each •	Illustration(s) for awarding each mark
7.	**(a) ans:** C(4,2) , $r = 5$ **2 marks** •1 For centre •2 For radius **(b) ans:** $4y + 3x = 45$ (or equiv.) **3 marks** •1 For gradient of radius •2 For gradient of tangent •3 For sub. using m_{tan} & T(7,6) **(c) ans:** S(-1,2) , P(-1,12) **3 marks** •1 For x_c - 5 = x_s = -1 , then coord. of S •2 For knowing to sub. x = -1 into equat. •3 For coordinates of P	(a) •1 C(4,2) •2 $r = \sqrt{4^2 + 2^2 - (-5)} = \sqrt{25} = 5$ (b) •1 $m_{CT} = \frac{6-2}{7-4} = \frac{4}{3}$ •2 $m_{\tan} = -\frac{3}{4}$ •3 $y - 6 = -\frac{3}{4}(x - 7)$ (c) •1 $4 - 5 = -1 \therefore S(-1,2)$ … same y as C •2 x-coordinate of P is same as S. Also on line $4y + 3x = 45$. $4y + 3(-1) = 45$ •3 $y = 12 \therefore P(-1,12)$
8.	**ans:** 90·0° **7 marks** •1 For re-arranging and realising … = 1 •2 For expansion & equating coefficients •3 Tan ratio •4 For α •5 For k •6 For solving to one-third •7 For angle	•1 $\sin x + \sqrt{8}\cos x = 1$ •2 $= k\cos x\cos\alpha + k\sin x\sin\alpha$ $k\cos\alpha = \sqrt{8}$, $k\sin\alpha = 1$ •3 $\tan\alpha = \frac{1}{\sqrt{8}}$ •4 $\alpha = 19 \cdot 5^\circ$ •5 $k = \sqrt{9} = 3$ •6 $\cos(x - 19 \cdot 5) = \frac{1}{3}$ •7 $x - 19 \cdot 5 = 70 \cdot 5 \therefore x = 90 \cdot 0^o$
9.	**(a) ans:** $k = 9$ **4 marks** •1 For knowing that at A the deriv. = 0 •2 For differentiating •3 For solving deriv = 0 and making x = 1 •4 Calculating k **(b) ans:** B(3,4) **3 marks** •1 Solving deriv. to zero when k = 9 •2 Finding x-coordinate of B •3 Sub. to find y-coordinate of B	(a) •1 A is a stat. point $\therefore \frac{dy}{dx} = 0$ *at A* •2 $\frac{dy}{dx} = 3x^2 - 12x + k$ •3 $3(1^2) - 12(1) + k = 0$ •4 $k = 9$ (b) •1 solve $3x^2 - 12x + 9 = 0$ •2 $3(x-1)(x-3) = 0 \Rightarrow x = 3$ for B •3 $y = 3^3 - 6(3^2) + 9(3) + 4 = 4$
		Total 67 marks