Higher Mathematics - Practice Examination F

Please note … the format of this practice examination is the same as the current format. The paper timings are the same, however, there are some differences in the marks allocated. Calculators may only be used in Paper 2.

MATHEMATICS

Higher Grade - Paper I (Non~calculator)

Time allowed - 1 hour 10 minutes

Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\ ,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(\ a\ ,\ b\ )$ and radius r.

Scalar Product: $a\ .\ b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a\ .\ b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad where \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad and \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned}
\sin\left(A \pm B\right) &= \sin A \cos B \pm \cos A \sin B \\
\cos\left(A \pm B\right) &= \cos A \cos B \mp \sin A \sin B \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A \\
\sin 2A &= 2\sin A \cos A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. Given that $x = -2$ and $x = 1$ are two roots of the equation $x^3 + px^2 - 6x + q = 0$, establish the values of p and q and hence find the third root of the equation. **(5)**

2. A sequence is defined by the recurrence relation $U_{n+1} = 0 \cdot 6U_n + 8$.

(a) Explain why this sequence has a limit as $n \to \infty$. **(1)**

(b) Find the limit of this sequence. **(2)**

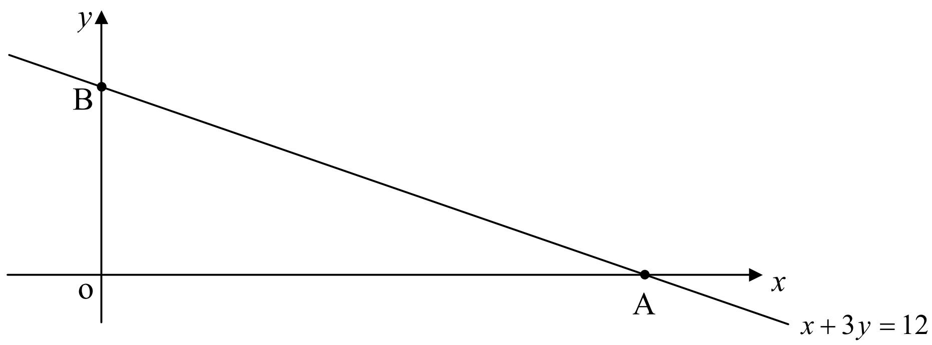
(c) Given that $L - U_1 = 3$, where L is the limit of this sequence, establish the value of U_0, the intitial value. **(3)**

3. A function is defined as $g(\theta) = 2\cos^2\theta - 2\cos 2\theta$.

Show that $g'(\theta)$ can be written in the form

$$g'(\theta) = 2\sin 2\theta$$ **(5)**

4. The line with equation $x + 3y = 12$ meets the x and the y axes at the points A and B respectively.



Find the equation of the perpendicular bisector of AB. **(5)**

5. Two functions f and g are defined on the set of real numbers as follows :

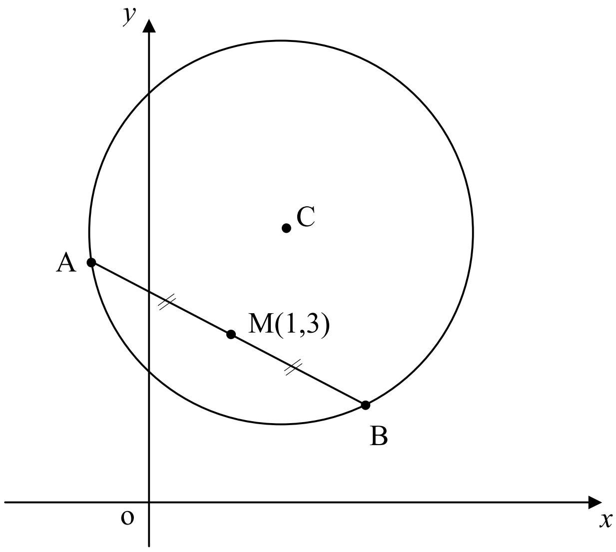
$$f(x) = 8-2x \quad , \quad g(x) = \tfrac{1}{2}(x+8).$$

(a) Evaluate $f\big(g(2)\big)$. **(1)**

(b) Find an expression , in its simplest form, for $g\left(f(x)\right)$. **(2)**

(c) Hence prove that $f^{-1}(x) = \tfrac{1}{2}\big[g\big(f(x)\big)\big]$. **(3)**

6. The circle below, centre C, has as its equation $x^2+y^2-4x-10y+19=0$.
M(1,3) is the mid-point of the chord AB.



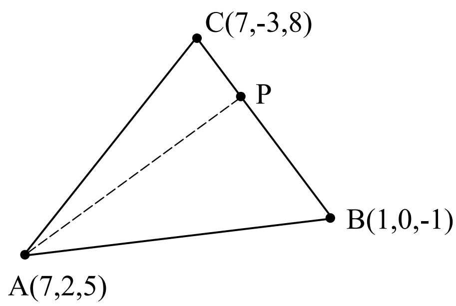
(a) Write down the coordinates of C, the centre of the circle. **(1)**

(b) Show that the equation of the chord AB can be written as $x = 7-2y$. **(3)**

(c) Hence find algebraically the coordinates of A and B.
(4)

7. Triangle ABC has as its vertices A(7,2,5), B(1,0,-1) and C(7,-3,8) as shown.

P is a point on side BC.



(a) Establish the coordinates of P given that BP = 2PC. **(3)**

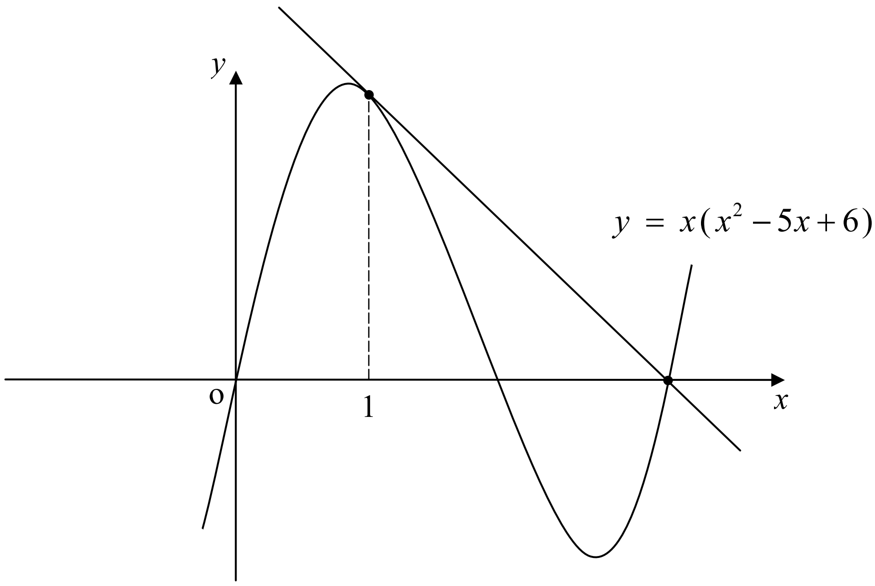
(b) Hence show that AP is perpendicular to BC. **(3)**

8. A radioactive substance decays according to the formula $M_t = M_o 20^{-0 \cdot 007t}$, where M_o is the intitial mass of the substance, M_t is the mass remaining after t years.

Calculate , to the nearest year, how long a sample would take to decay to **half** its original mass. **(4)**

9. Part of the graph of the curve $y = x(x^2 - 5x + 6)$ is shown in the diagram.

The tangent to the curve at the point where $x = 1$ is also shown.



(a) Find the equation of the tangent to the curve at the point where $x = 1$. **(4)**

(b) Show that this tangent also passes through one of the points where the curve crosses the x-axis. **(2)**

[END OF QUESTION PAPER]

<table>
<tr><th></th><th>Give 1 mark for each •</th><th>Illustration(s) for awarding each mark</th></tr>
<tr><td>1.</td><td>ans: p = -3 , q = 8 : x = 4 5 marks
•1 Setting up synth. division
•2 Obtaining first equation
•3 Obtaining second equation
•4 Solving system for p and q
•5 Sub. (say p in quotient) for 3rd root</td><td>•1 $1 \,|\, 1 \quad p \quad -6 \quad q$
•2 $p+q=5$
•3 $4p+q=-4$
•4 $p=-3$, $q=8$
•5 $x^2-2x-8=0 \Rightarrow (x+2)(x-4)=0$
$x=4$ is missing root</td></tr>
<tr><td>2.</td><td>(a) ans: statement 1 mark
•1 Explanation
(b) ans: 20 2 marks
•1 Method used
•2 For calculating limit
(c) ans: $U_0 = 15$ 3 marks
•1 For initial equating and finding U_1
•2 For recurrence with U_1 in place
•3 For answer</td><td>(a) •1 Because $-1 < a < 1$ (or equiv.)
(b) •1 $L=\frac{b}{1-a}$
•2 $L=\frac{8}{1-0\cdot 6}=\frac{8}{0\cdot 4}=\frac{80}{4}=20$
(c) •1 $20-U_1=3 \;\therefore\; U_1=17$
•2 $17=0\cdot 6U_0+8$
•3 $9=0\cdot 6U_0 \Rightarrow U_0=\frac{9}{0\cdot 6}=\frac{90}{6}$
$U_0=15$</td></tr>
<tr><td>3.</td><td>ans: proof 5 marks
•1 For diff. power in first term
•2 For diff. $\cos\theta$ in first term
•3 For differentiating second term
•4 For extracting $2\sin\theta\cos\theta$ for replace.
•5 Simplifying to given answer</td><td>•1 .... $(4\cos\theta)$ ....
•2 $-\sin\theta$ i.e. $-4\cos\theta\sin\theta$
•3 .... $(-4\sin 2\theta)$
•4 $-2(2\sin\theta\cos\theta)-(-4\sin 2\theta)$
•5 $-2\sin 2\theta+4\sin 2\theta=2\sin 2\theta$</td></tr>
<tr><td>4.</td><td>ans: $y=3x-16$ 5 marks
•1 For points A & B
•2 Gradient of AB
•3 Gradient of perpendicular
•4 For mid-point of AB
•5 For equation of perpen. bisector</td><td>•1 A(12,0) , B(0,4)
•2 $m_{AB}=-\frac{1}{3}$
•3 $m_{per.}=3$
•4 M(6,2)
•5 $y-2=3(x-6)$</td></tr>
<tr><td>5.</td><td>(a) ans: $f(g(2))=-2$ 1 mark
•1 For answer
(b) ans: $g(f(x))=8-x$ 2 marks
•1 For substitution
•2 For simplifying</td><td>(a) •1 $f(g(2))=-2$
(b) •1 $g(8-2x)=\frac{1}{2}\left((8-2x)+8\right)$ (or equiv.)
•2 $g(f(x))=\frac{1}{2}(16-2x)$
$= 8-x$</td></tr>
</table>

	Give 1 mark for each •	Illustration(s) for awarding each mark
5.	**(c) ans: proof** **3 marks** •1 For knowing how to find inverse •2 For finding inverse •3 For final statement (equating)	•1 $To\ find\ \ f^{-1}(x) \Rightarrow y = 8 - 2x$ •2 $2x = 8 - y \Rightarrow x = \frac{1}{2}(8 - y)$ •3 $f^{-1}(x) = \frac{1}{2}(8 - x) = \frac{1}{2}[g(f(x))]$
6.	**(a) ans: C(2,5)** **1 mark** •1 For answer **(b) ans: proof** **3 marks** •1 For gradient of CM •2 For perp. gradient of chord AB •3 For equ. of chord and rearranging **(c) ans: A(-1,4) , B(3,2)** **4 marks** •1 For attempting to solve a system •2 For expanding and simplifying •3 For factorising and finding y coords. •4 For completing points	(a) •1 C(2,5) (b) •1 $M_{cm} = \frac{5-3}{2-1} = 2$ •2 $M_{AB} = -\frac{1}{2}$ •3 $y - 3 = -\frac{1}{2}(x - 1)$ $2y - 6 = -x + 1$ $x = 7 - 2y$ (c) •1 $(7 - 2y)^2 + y^2 - 4(7 - 2y) - 10y + 19 = 0$ •2 $5y^2 - 30y + 40 = 0$ •3 $(5(y - 4)(y - 2) - 0 \ \therefore\ y = 4\ ,\ y = 2$ •4 $y = 4\ then\ x = -1\ ,\ y = 2\ when\ x = 3$
7.	**(a) ans: P(5,-2,5)** **3 marks** •1 For initial vector algebra •2 For simplification •3 Answer **(b) ans: proof** **3 marks** •1 Scalar product statement (or implied) •2 For vectors in component form •3 Scalar product calculation to zero	(a) •1 $\underset{\sim}{p} - \underset{\sim}{b} = 2(\underset{\sim}{c} - \underset{\sim}{p})$ •2 $3\underset{\sim}{p} = 2\underset{\sim}{c} + \underset{\sim}{b}$ •3 P(5,-2,5) (b) •1 If perp. then $\overrightarrow{AP}.\overrightarrow{BC} = 0$ •2 $\overrightarrow{AP} = \begin{pmatrix} -2 \\ -4 \\ 0 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix}$ •3 $-12 + 12 + 0 = 0 \ \therefore\ perp.$
8.	**ans: 33 years** **4 marks** •1 For solving to 0.5 •2 For taking logs •3 For releasing the power •4 calculation to answer	•1 $20^{-0\cdot007t} = 0\cdot5$ •2 $\log 20^{-0\cdot007t} = \log 0\cdot5$ •3 $-0\cdot007t \log 20 = \log 0\cdot5$ •4 $-0\cdot007t = \frac{\log 0\cdot5}{\log 20} \quad \therefore \quad t = 33$
9.	**(a) ans:** $y = -x + 3$ **4 marks** •1 Completing point of tangency •2 Differentiating to find m •3 Finding gradient of tangent •4 Point + m in equation **(b) ans: proof** **2 marks** •1 Finding where tan. cuts x-axis •2 Showing that point satisfies equation	(a) •1 $y = 1(1 - 5 + 6) = 2 \ \therefore\ T(1,2)$ •2 $\frac{dy}{dx} = m = 3x^2 - 10x + 6$ •3 $m = 3(1^2) - 10(1) + 6 = -1$ •4 $y - 2 = -1(x - 1)$ various methods .... (b) •1 When $y = 0$ then $x = 3$ •2 Sub $x = 3$ in equ. of curve $y = 3(3^2 - 5(3) + 6) = 3(0) = 0$

Total 51 marks

Higher Mathematics - Practice Examination F

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MATHEMATICS

Higher Grade - Paper II

Time allowed - 1 hour 30 minutes

Read Carefully

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FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g\ ,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

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Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\sin\left(A \pm B\right) = \sin A \cos B \pm \cos A \sin B$$

$$\cos\left(A \pm B\right) = \cos A \cos B \mp \sin A \sin B$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

$$\sin 2A = 2\sin A \cos A$$

Table of standard derivatives:

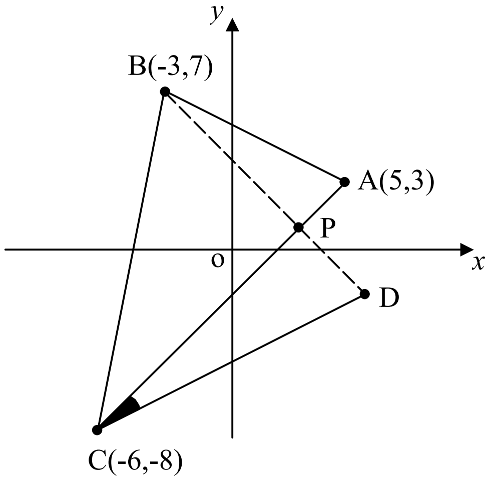
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. Triangle ABC has vertices A(5,3), B(-3,7) and C(-6,-8) as shown.
The altitude through B meets AC at P.



(a) Find the equation of side AC and the equation of the altitude BP. **(4)**

(b) Hence find the coordinates of P. **(3)**

(c) BP is produced in such a way that PD = $\frac{1}{2}$ BP. Establish the coordinates of D. **(1)**

(d) By considering gradients, calculate the size of angle DCP to the nearest degree. **(3)**

2. Solve algebraically the equation

$$\sin x^\circ - 3\cos 2x^\circ + 2 = 0, \qquad 0 \leq x < 360.$$ **(5)**

3. Two functions are defined as $f(x) = (x+2)(x+1)$ and $g(x) = x(x-2)$.

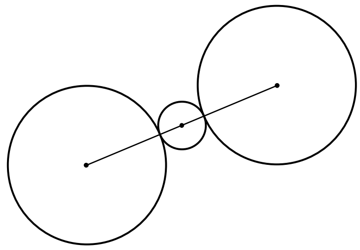
(a) Given that $h(x) = f\big(g(x)\big)$, show clearly that $h(x) = x^4 - 4x^3 + 7x^2 - 6x + 2$. **(3)**

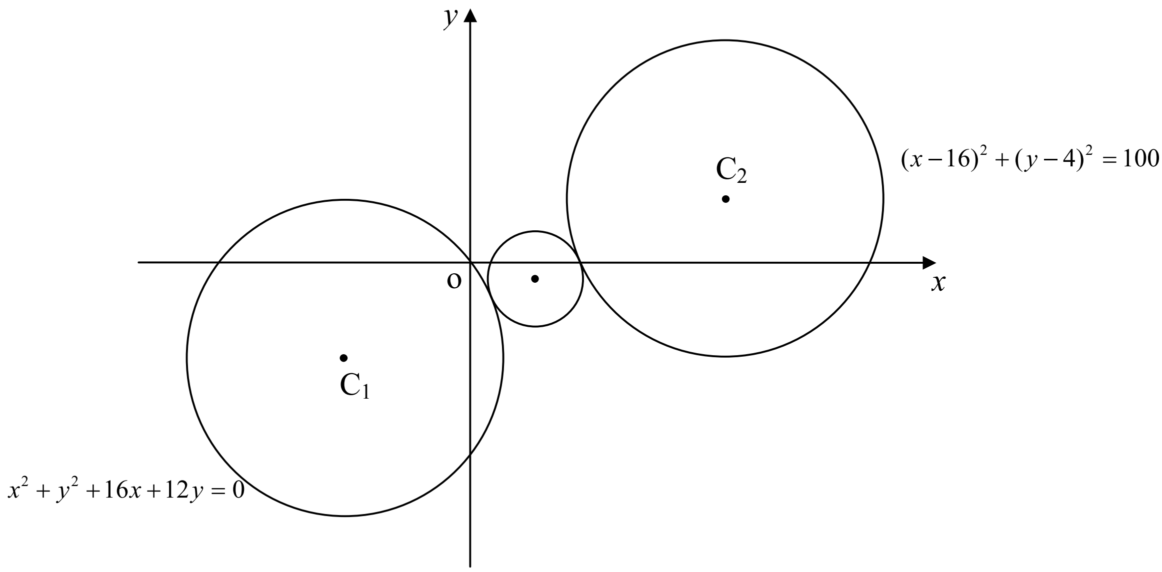
(b) Hence solve the equation $h(x) = 0$ showing that it has, in fact, only one real root. **(4)**

4. Three wheels are positioned in such a way that their centres are collinear.

When placed on a set of rectangular axes the equations of the two larger circles are

$x^2 + y^2 + 16x + 12y = 0$ and $(x-16)^2 + (y-4)^2 = 100$, as shown.





(a) Write down the coordinates of the two centres C_1 and C_2. **(2)**

(b) Calculate the radii of the two larger circles and the distance between the two centres C_1 and C_2. **(4)**

(c) Hence establish the centre and radius of the small circle and write down its equation. **(3)**

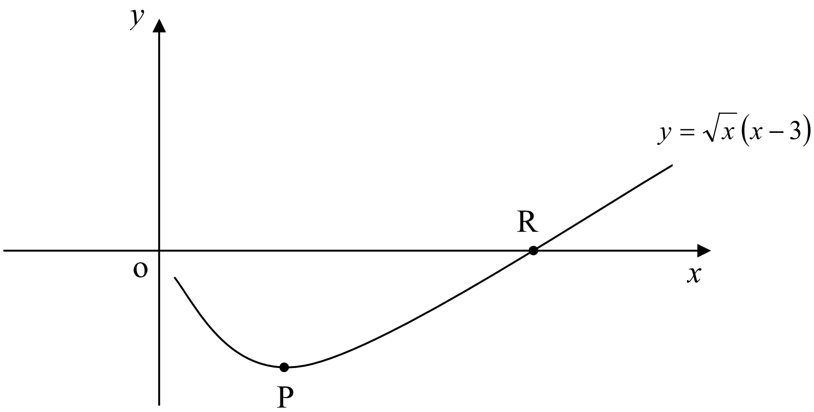
5. Two vectors are defined as $F_1 = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $F_2 = p\underset{\sim}{i} - \sqrt{2}\,\underset{\sim}{j} + \sqrt{3}\,\underset{\sim}{k}$.

(a) Given that these two vectors have the same magnitude, find the value of p, where $p > 0$. **(3)**

(b) Hence calculate the angle between these two vectors, giving your answer correct to one decimal place. **(3)**

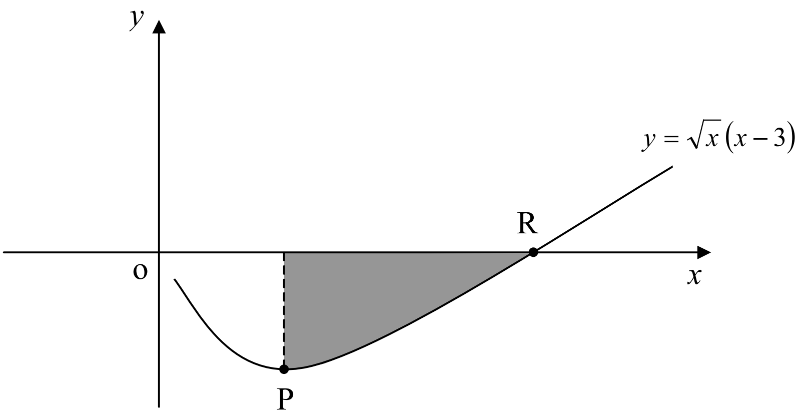
6. A function is defined as $f(x) = \sqrt{x}\,(x-3)$, where only the positive value of $\sqrt{x}$ is taken for each value of $x > 0$.

Part of the graph of $y = f(x)$ is shown below.



(a) Find the coordinates of the turning point at P and the root at R. **(6)**

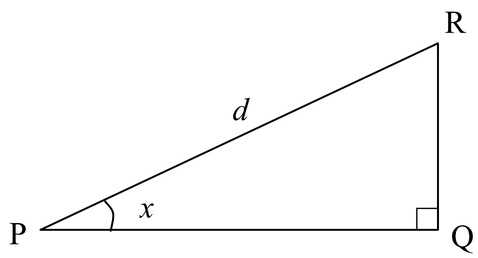
(b) Hence calculate the shaded area below giving your answer **correct to 2 decimal places**.



(5)

7. Triangle PQR is right-angled at Q.

The hypotenuse is d units long and angle QPR = x radians.



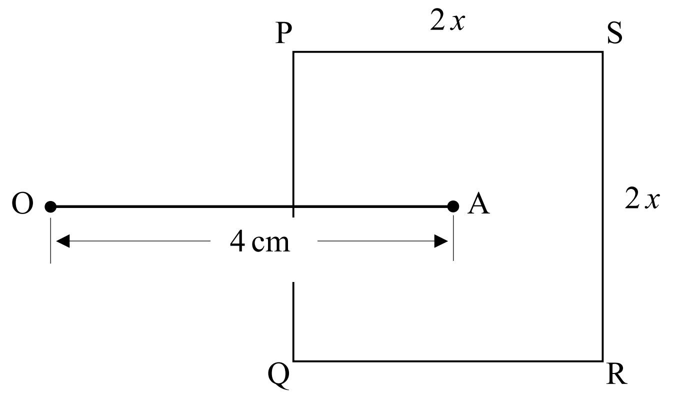
(a) Explain why PQ = $d\cos x$ and QR = $d\sin x$ units long. **(2)**

(b) Hence show that the perimeter (P) of the triangle can be expressed in the form

$$P = d + \sqrt{2}\,d\cos(x - \tfrac{\pi}{4})$$ **(7)**

8. In the diagram below PQRS is a square of side $2x$ cm.

A straight line OA, measuring 4 cm, has been drawn in such a way that A lies at the centre of the square and OA is parallel to PS.



(a) Show that $\mathrm{OP}^2 = 2x^2 - 8x + 16$. **(4)**

(b) Hence, by completing the square, or otherwise, find x for which the length of OP is at a minimum and state the minimum length of OP. **(4)**

[END OF QUESTION PAPER]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**(a) ans:** $y = x - 2$, $y = -x + 4$ **4 marks** •1 For gradient of AC •2 For equation of AC •3 For finding gradient of altitude •4 For equation of altitude **(b) ans:** P(3,1) **3 marks** •1 Knowing to solve a system •2 Finding first coordinate •3 Finding second coord. **(c) ans:** D(6,-2) **1 mark** •1 Answer **(d) ans:** 18° **3 marks** •1 For knowing and using $\tan\theta = m$ •2 For angle between CD and horoz. •3 For 45^{o} and subtraction to ans.	(a) •1 $m_{AC} = \frac{3+8}{5+6} = 1$ •2 $y - 3 = 1(x - 5)$ •3 $m_{alt} = -1$ •4 $y - 7 = -1(x + 3)$ (b) •1 $x - 2 = -x + 4$ •2 $2x = 6 \Rightarrow x = 3$ •3 $y = 3 - 2 = 1$ (c) •1 $\rightarrow 6 \downarrow 6 \therefore \rightarrow 3 \downarrow 3$ from P, D(6,-2) (d) •1 $m_{AC} = 1 \therefore \tan^{-1} 1 = \theta = 45^\circ$ •2 $m_{CD} = \frac{-2+8}{6+6} = 0\cdot5 \therefore \tan^{-1} 0\cdot5 = 26\cdot6^\circ$ •3 $45 - 26\cdot6 = 18^\circ$
2.	**ans:** $\{\, 19\cdot5 , 160\cdot5 , 210 , 330 \,\}$ **5 marks** •1 For correct substitution •2 For re-arranging to quadratic •3 Factorising to two roots •4 Two ans. from one root •5 Two ans. from second root	•1 $\sin x - 3(1 - 2\sin^2 x) + 2 = 0$ •2 $6\sin^2 x + \sin x - 1 = 0$ •3 $\sin x = \frac{1}{3}$ *or* $\sin x = -\frac{1}{2}$. •4 $19\cdot5^o$, $160\cdot5^o$ •5 210^o , 330^o
3.	**(a) ans:** proof **3 marks** •1 For expanding original functions •2 For correct substitution •3 For expanding to answer **(b) ans:** x = 1 , proof **4 marks** •1 Knowing to use synthetic division •2 Finding the root , $x = 1$ •3 Using x = 1 again ! •4 Showing remaining quotient has no roots	(a) •1 $f(x) = x^2 + 3x + 2$, $g(x) = x^2 - 2x$ •2 $h(x) = (x^2 - 2x)^2 + 3(x^2 - 2x) + 2$ •3 $h(x) = x^4 - 4x^3 + 4x^2 + 3x^2 - 6x + 2$ (b) •1 $\begin{array}{\|rrrrr} 1 & -4 & 7 & -6 & 2 \\ \hline \end{array}$ •2 $\begin{array}{r\|rrrrr} 1 & 1 & -4 & 7 & -6 & 2 \\ & & 1 & -3 & 4 & -2 \\ \hline & 1 & -3 & 4 & -2 & 0 \end{array}$ •3 1 again leaves quot. $x^2 - 2x + 2$ •4 for $b^2 - 4ac = -4 \therefore$ no more roots
4.	**(a) ans:** C_1(-8,-6) , C_2(16,4) **2 marks** •1 For first centre •2 For second centre **(b) ans:** $r_1 = 10$, $r_2 = 10$, $d = 26$ **4 marks** •1 Finding r of C_1 •2 Finding r of C_2 •3 For method (dist. form , pyth, etc.) •4 For correct distance	(a) •1 C_1(-8,-6) •2 C_2(16,4) (b) •1 $r = \sqrt{(-8)^2 + (-6)^2 - 0} = \sqrt{100} = 10$ •2 $r = \sqrt{100} = 10$ •3 $d = \sqrt{(x_2 - x_1)^2 \text{.......}}$ etc. •4 $d = \sqrt{676} = 26$

	Give 1 mark for each •	Illustration(s) for awarding each mark
4.	**(c) ans:** $C_3(4,-1)$, $r=3$ **3 marks** $(x-4)^2+(y+1)^2=9$ •1 For centre •2 For radius •3 For sub. into equ. to answer	(c) •1 Centre must be mid-pt $C_3(4,-1)$ •2 $r=(26-20)\div 2=3$ •3 $(x-4)^2+(y+1)^2=9$ or $x^2+y^2-8x+2y+8=0$
5.	**(a) ans:** $p=2$ **3 marks** •1 For calculating magnitude of F_1 •2 For equating second magnitude •3 Answer **(b) ans:** $\theta=93\cdot6^\circ$ **3 marks** •1 For scalar product of F_1 and F_2 •2 For $\cos\theta=$ .... •3 Answer	(a) •1 $\lvert F_1\rvert=\sqrt{2^2+2^2+(-1)^2}=3$ •2 $\lvert F_2\rvert$ then $p^2+2+3=9$ (or equiv.) •3 $p=2$ (b) •1 $F_1.F_2=\begin{pmatrix}2\\2\\-1\end{pmatrix}.\begin{pmatrix}2\\-\sqrt{2}\\\sqrt{3}\end{pmatrix}=4-2\sqrt{2}-\sqrt{3}$ •2 $\cos\theta=\dfrac{4-2\sqrt{2}-\sqrt{3}}{3\times3}$ •3 $\theta=93\cdot6^\circ$
6.	**(a) ans:** P(1,-2) , R(3,0) **6 marks** •1 Preparing to differentiate •2 Knowing to solve deriv. to zero •3 Differentiating •4 Solving to answer for x coord. of P •5 Finding y coord. of P •6 Finding root (coords. of R) **(b) ans:** Area = $2\cdot55$ units^2 **5 marks** •1 For setting up correct integral •2 For integrating first term •3 Integrating 2nd term •4 Substituting limits •5 Calculations to answer	(a) •1 $f(x)=x^{\frac32}-3x^{\frac12}$ •2 S.P. when $f'(x)=0$ (stated or impl.) •3 $f'(x)=\frac32x^{\frac12}-\frac32x^{-\frac12}$ (or equivalent) •4 $\dfrac{3\sqrt{x}}{2}-\dfrac{3}{2\sqrt{x}}=0$ $(\times\ 2\sqrt{x})$ $3x-3=0\ \therefore\ x=1$ •5 $y=\sqrt{1}(1-3)=-2$ •6 $x-3=0$, $x=3$ (b) •1 A = $-\displaystyle\int_1^3 x^{\frac32}-3x^{\frac12}\ dx$ •2 .... $\frac25x^{\frac52}$ (or equivalent) •3 ..... $2x^{\frac32}$ (or equivalent) •4 A = $-\left[\frac25(3^{\frac52})-2(3^{\frac32})\right]-\left[\frac25-2\right]$ •5 A = $-[6\cdot24-10\cdot39]-[-1\cdot6]=2\cdot56$
7.	**(a) ans:** proof **2 marks** •1 For knowing to involve trig ratios •2 For manipulation **(b) ans:** proof **7 marks** •1 Writing down an expression for P •2 Know to involve $k\cos(x-\alpha)$ •3 Select expansion and expand •4 Compare coefficients •5 Find k •6 Interpret comparison and find angle •7 Manipulation to answer	(a) •1 $\sin x=\frac{o}{d}\Rightarrow$ ... etc. •2 $QR=d\sin x$ *and* $PQ=d\cos x$ (b) •1 $P=d+d\cos x+d\sin x$ •2 $P=d+d(\cos x+\sin x)$ •3 $\cos x+\sin x=k\cos(x-\alpha)$ $=k\cos x\cos\alpha+k\sin x\sin\alpha$ •4 $k\sin\alpha=1$, $k\cos\alpha=1$ •5 $k=\sqrt{1^2+1^2}=\sqrt{2}$ •6 $\tan\alpha=\dfrac11=1\ \therefore\ \alpha=\pi/4$ •7 $P=d+d[\sqrt{2}\cos(x-\frac{\pi}{4})]$

	Give 1 mark for each •	Illustration(s) for awarding each mark
8.	**(a) ans:** proof **4 marks** •1 For attempting to use pythagoras •2 For length x •3 For length (4-x) •4 For expansion to answer **(b) ans:** $x = 2$, $OP_{min} = \sqrt{8}$ **4 marks** •1 For removing common factor •2 Completing the square with $x^2 - 4x$ •3 Tidying to final form •4 Answer for replacement and minimum *(discretion for minimum* 8 *instead of* $\sqrt{8}$*)*	(a) •1 $OP^2 = a^2 + b^2$ (stated or implied) •2 $OP^2 = x^2 + \ldots\ldots\ldots\ldots$ •3 $OP^2 = \ldots\ldots\ldots + (4-x)^2$ •4 $OP^2 = x^2 + 16 - 8x + x^2$ $= 2x^2 - 8x + 16$ (b) •1 $2(x^2 - 4x) + 16$ •2 $[(x-2)^2 - 4]$ •3 $OP^2 = 2(x-2)^2 + 8$ •4 minimum when x = 2 minimum value of $OP^2 = 8$ $\therefore OP_{min} = \sqrt{8}$ *NB... Pupils may use differentiation to answer this part of the question .... assign marks accordingly.*
		Total 66 marks