Chapter 3 Review page 93

1.(a) $\int \frac{2x - x^3}{x^{\frac{3}{2}}} dx = \int \frac{2x}{x^{\frac{3}{2}}} - \int \frac{x^3}{x^{\frac{3}{2}}} = \int 2x^{-\frac{1}{2}} - x^{\frac{3}{2}}$

$= \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \boxed{4x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C}$

(b) $\int \cos(4x-1)\,dx = \boxed{\frac{\sin(4x-1)}{4} + C}$

(c) $\int (2x+3)^5 dx = \frac{(2x+3)^6}{6(2)} = \boxed{\frac{1}{12}(2x+3)^6 + C}$

(d) $\int (9x^4 - 6x^2 + 1)\,dx = \frac{9x^5}{5} - \frac{6x^3}{3} + x = \boxed{\frac{9}{5}x^5 - 2x^3 + x + C}$

(e) $\int (\sec^2 3x)\,dx = \frac{\tan 3x}{3} + C = \boxed{\frac{1}{3}\tan 3x + C}$

(f) $\int (e^{4x-3})\,dx = \frac{1}{4}\int (4e^{4x-3})\,dx = \boxed{\frac{1}{4}e^{4x-3} + C}$

(g) $\int \left(\frac{4}{x}\right) dx = 4\int \frac{1}{x}\,dx = \boxed{4\ln|x| + C}$

(h) $\int \frac{1}{(1-2\sin^2 x)^2}\,dx = \int \frac{1}{(\cos 2x)^2} = \int \sec^2 2x\,dx$

$= \frac{\tan 2x}{2} + C = \boxed{\frac{1}{2}\tan 2x + C}$

2. (a) $\int (x^2+2)\left(\frac{1}{3}x^3 + 2x\right)\frac{dx}{du}\,du$

$= \int (x^2+2) \cdot u \cdot \frac{1}{(x^2+2)}\,du$

$= \int u\,du = \frac{u^2}{2} + C$

let $u = \frac{1}{3}x^3 + 2x$

$\frac{du}{dx} = x^2 + 2$

$\frac{dx}{du} = \frac{1}{x^2+2}$

$= \frac{\left(\frac{1}{3}x^3 + 2x\right)^2}{2} + C = \boxed{\frac{1}{2}\left(\frac{1}{3}x^3 + 2x\right)^2 + C}$

2 (b) $\int (-2\sin 2x\, e^{\cos 2x}) \frac{dx}{du} du$

$= \int -2\sin 2x\, e^{u} \cdot \frac{1}{-2\sin 2x} du$

$= \int e^{u} du = e^{u} + C = \boxed{e^{\cos 2x} + C}$

let $u = \cos 2x$

$\frac{du}{dx} = -\sin 2x \cdot 2$

$\frac{dx}{du} = -\frac{1}{2\sin 2x}$

3. $\int (x+3) \sqrt[3]{x^2+6x-1}\, \frac{dx}{dt} dt$

$= \int (x+3)\, t^{\frac{1}{3}} \cdot \frac{1}{2(x+3)} dt$

$= \int \frac{t^{\frac{1}{3}}}{2} dt = \frac{3t^{\frac{4}{3}}}{2 \cdot 4} = \frac{3t^{\frac{4}{3}}}{8} + C$

$= \boxed{\frac{3}{8}(x^2+6x-1)^{\frac{4}{3}} + C}$

let $t = x^2+6x-1$

$\frac{dt}{dx} = 2x+6$

$\frac{dx}{dt} = \frac{1}{2(x+3)}$

4. $\int \frac{1}{16+x^2} dx$

$= \int \frac{1}{16+16\tan^2 t} \cdot 4\sec^2 t\, dt$

$= \frac{1}{4} \int \frac{1}{1+\tan^2 t} \cdot \sec^2 t\, dt \Rightarrow \{ \sec^2 t = 1 + \tan^2 t \}$

$= \frac{1}{4} \int \frac{1+\tan^2 t}{1+\tan^2 t} dt = \frac{1}{4} \int 1\, dt = \frac{1}{4} t + C$

$\{$because $x = 4\tan t \Rightarrow \tan t = \frac{x}{4} \Rightarrow \underline{t = \tan^{-1}(\frac{x}{4})} \}$

$= \frac{1}{4} \cdot \tan^{-1}\left(\frac{x}{4}\right) + C$ or $\boxed{\frac{1}{4} \tan^{-1}\left(\frac{x}{4}\right) + C}$

let $x = 4\tan t$

$\frac{dx}{dt} = 4\sec^2 t\, dt$

$x^2 = 16\tan^2 t$

5. $\int_0^1 4x \sqrt[3]{1-x^2}\, dx$

$= \int 4(1-t^2)^{\frac{1}{2}} (t^2)^{\frac{1}{3}} \cdot \frac{-t}{(1-t^2)^{\frac{1}{2}}}$

$= -4\int t^{\frac{2}{3}} \cdot t = -4\int t^{\frac{5}{3}} dt$

$= 3\frac{t^{\frac{8}{3}}}{8} = -4\left[\frac{3}{8}\left(\left(1-x^2\right)^{\frac{1}{2}}\right)^{\frac{8}{3}}\right]_0^1$

$= -4\left[\frac{3}{8}(1-x^2)^{\frac{4}{3}}\right]_0^1 = -4\left(0-\frac{3}{8}(1)^{\frac{4}{3}}\right) = \frac{12}{8} = \boxed{\frac{3}{2}}$ ✓

let $t^2 = 1-x^2$

$t = (1-x^2)^{\frac{1}{2}}$

$x^2 = 1-t^2$

* $x = (1-t^2)^{\frac{1}{2}}$

$\frac{dx}{dt} = \frac{1}{2}(1-t^2)^{2}(-2t)$

* $\frac{dx}{dt} = -t(1-t^2)^{-\frac{1}{2}}$

6. $\int_1^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\, dx$

$= \int \frac{1}{\sqrt{4-4\sin^2 t}} \cdot 2\cos t$

$= \int \frac{1}{\sqrt{4(1-\sin^2 t)}} \cdot 2\cos t$

$= \int \frac{1}{\sqrt{1-\sin^2 t}} \cdot \cos t$

let $x = 2\sin t$

$x^2 = 4\sin^2 t$

$\frac{dx}{dt} = 2\cos t$

$x = 2\sin t$

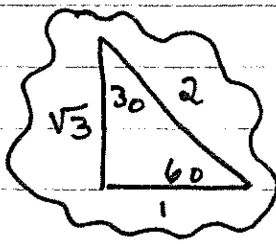
$\sin t = \frac{x}{2}$

$\boxed{t = \sin^{-1}\left(\frac{x}{2}\right)}$

$= \int \frac{\cos t}{\sqrt{\cos^2 t}}\, dt = \int 1\, dt = t$

$60° = \frac{\pi}{3}$

$30° = \frac{\pi}{6}$



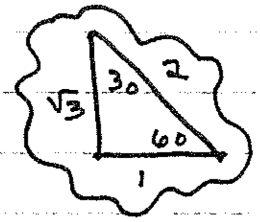
$= \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_1^{\sqrt{3}}$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} - \frac{\pi}{6} = \boxed{\frac{\pi}{6}}$ ✓

7. $\displaystyle\int_a^{2a} \frac{4}{x}\,dx = \Big[4 \ln|x|\Big]_a^{2a} = 4(\ln 2a - \ln a)$

$= 4 \ln \frac{2a}{a} = \boxed{4 \ln 2}$ or $\ln 2^4 = \underline{\ln 16}$

8. $\displaystyle\int_{\pi/6}^{\pi/3} \frac{1}{\csc^2 x - 1}\,dx$ $\left\{ \text{show} = \frac{3-1}{\sqrt{3}} - \frac{\pi}{6} \right\}$



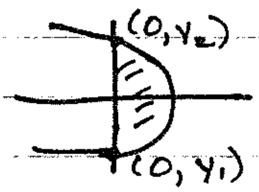
$= \displaystyle\int \frac{1}{\cot^2 x}\,dx = \int \tan^2 x\,dx = \int \sec^2 - 1\,dx$

$= \Big[\tan x - x\Big]_{\pi/6}^{\pi/3} = \tan\frac{\pi}{3} - \frac{\pi}{3} - \left(\tan\frac{\pi}{6} - \frac{\pi}{6}\right)$

$= \sqrt{3} - \frac{\pi}{3} - \frac{1}{\sqrt{3}} + \frac{\pi}{6} = \frac{3}{\sqrt{3}} - \frac{2\pi}{6} - \frac{1}{\sqrt{3}} + \frac{\pi}{6}$

$= \boxed{\frac{3-1}{\sqrt{3}} - \frac{\pi}{6}}$ ✓ or $\underline{\frac{2}{\sqrt{3}} - \frac{\pi}{6}}$

9. $y^2 = 4 - x$



{calculate shaded area.}

let $x = 0$

$y^2 = 4$ then $\boxed{y = -2 \text{ or } y = 2}$

if $y^2 = 4 - x$ then $\underline{x = 4 - y^2}$

$\displaystyle\int_{-2}^{2} (4 - y^2)\,dy = \left[4y - \frac{y^3}{3}\right]_{-2}^{2} = 4(2) - \frac{2^3}{3} - \left(4(-2) - \frac{(-2)^3}{3}\right)$

$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = 16 - \frac{16}{3} = \frac{48}{3} - \frac{16}{3} = \boxed{\frac{32}{3}}$

10. $a = 6t + 2$

$v = \displaystyle\int (6t + 2)\,dt = \frac{6t^2}{2} + 2t = \boxed{3t^2 + 2t} \Leftarrow \underline{v}$

$s = \displaystyle\int (3t^2 + 2t)\,dt = \frac{3t^3}{3} + \frac{2t^2}{2} = \boxed{t^3 + t^2} \Leftarrow \underline{s}$

at 3 sec. $V = 3(3)^2 + 2(3) = \boxed{33 \text{ m/s}}$

at 3 sec $S = 3^3 + 3^2 = \boxed{36 \text{ m}}$

11. $y = x^3$ find volume of revolution from $y=0$ to $y=8$

$\hookrightarrow x = y^{\frac{1}{3}}$

$$\pi \int_0^8 \left(y^{\frac{1}{3}}\right)^2 dy = \int_0^8 y^{\frac{2}{3}} dy$$

$$= \pi \left[\frac{3y^{\frac{5}{3}}}{5} \right]_0^8 = \frac{\pi \cdot 3(8)^{\frac{5}{3}}}{5} = \frac{\pi \cdot 3 \cdot 32}{5} = \boxed{\frac{96\pi}{5}}$$