Chapter 2.1 Review page 48

1. Find the derivative of $4x^2 - 3x$ from First Principles.

$f(x) = 4x^2 - 3x \qquad f(x+h) = 4(x+h)^2 - 3(x+h)$

$$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2 - 3(x+h) - 4x^2 + 3x}{h}$$

$$\frac{4(x^2 + 2hx + h^2) - 3x - 3h - 4x^2 + 3x}{h}$$

$$\frac{\cancel{4x^2} + 8hx + 4h^2 - \cancel{3x} - 3h - \cancel{4x^2} + \cancel{3x}}{h}$$

$$\frac{4h^2 + 8hx - 3h}{h} = 4h + 8x - 3$$

$$= \lim_{h \to 0} 4(0) + 8x - 3 = \boxed{8x - 3}$$

2. find $\frac{dy}{dx}$ $\frac{4x^5 - 3x}{2x^2}$ {Quotient Rule}

$$\frac{dy}{dx} = \frac{(20x^4 - 3)(2x^2) - (4x^5 - 3x)(4x)}{(2x^2)^2}$$

$$= \frac{40x^6 - 6x^2 - 16x^6 + 12x^2}{4x^4}$$

$$= \frac{24x^6 + 6x^2}{4x^4} = \boxed{\frac{12x^4 + 3}{2x^2}} \ \text{OR} \equiv \boxed{6x^2 + \frac{3}{2x^2}}$$

3. $f(x) = \cos 2x$, find $f'\left(\frac{2\pi}{3}\right)$

$f'(x) = -2\sin 2x$

$= -2\sin\left(\frac{4\pi}{3}\right) = \sqrt{3}$ {use calculator in Radians}

4. find $\frac{dy}{dx}$: $\sqrt{x^3}\cdot\sin x = x^{\frac{3}{2}}\sin x$ {Product Rule}

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}\sin x + x^{\frac{3}{2}}\cdot\cos x$ {simplify}

$= x^{\frac{1}{2}}\left(\frac{3}{2}\sin x + x\cos x\right)$

$= \boxed{\sqrt{x}\left(\frac{3}{2}\sin x + x\cos x\right)}$

5. find $\frac{dy}{x}$ $\frac{\cos x}{(3x-2)^2}$ {Quotient Rule}

$= \frac{-\sin x(3x-2)^2 - \cos x\cdot 2(3x-2)(3)}{(3x-2)^4}$ {tidy up}

$= \boxed{\frac{-(3x-2)\sin x - 6\cos x}{(3x-2)^3}}$

6. $f(x) = \cot 3x$, find $f'(x)$ {cosec ← −cosec ← cotan}

$\boxed{f'(x) = -3\,\mathrm{cosec}^2(3x)}$

{a longer way of finding this is to rewrite:
$\cot 3x = \frac{1}{\tan 3x} = \frac{1}{\frac{\sin 3x}{\cos 3x}} = \frac{\cos 3x}{\sin 3x}$ and using Quotient Rule}

7. $y = \sec x \tan x$ find $\frac{dy}{dx}$ $\left\{ \begin{matrix} \sec \to \sec \to \tan \\ \text{cosec} \quad -\text{cosec} \quad \text{cotan} \end{matrix} \right\}$

$\frac{dy}{dx} = \sec x \cdot \tan x \cdot \tan x + \sec x (\sec^2 x)$

$\boxed{\frac{dy}{dx} = \sec x ( \tan^2 x + \sec^2 x)}$

tricky: trig identity $\left\{ \begin{matrix} \tan^2 x = \sec^2 x - 1 \\ \text{(substitution)} \end{matrix} \right.$

$\frac{dy}{dx} = \sec x (\underline{\sec^2 x - 1} + \sec^2 x)$

$\boxed{\frac{dy}{dx} = \sec x ( 2 \sec^2 x - 1)}$ ← book answer! I don't think its necessary to take it this far

8. $y = \text{cosec}^3 x$, show $\frac{dy}{dx} + 3y \cot x = 0$

$y = (\text{cosec } x)^3$

$\frac{dy}{dx} = 3(\text{cosec } x)^2 (-\text{cosec } x)(\text{cotan } x)$

$\left\{ \begin{matrix} \frac{dy}{dx} = -3 \text{cosec}^3 x \cdot \text{cotan } x \\ 3y \cot x = 3(\text{cosec}^3 x) \text{ cotan } x \end{matrix} \right.$

by substitution:

$\frac{dy}{dx} + 3y \cot x = -3 \text{cosec}^3 x \cdot \cot x + 3 \text{cosec}^3 x \text{ cotan } x$

$= \underline{\underline{0}}$

9. $y = x^2 \ln x$ {product rule}

$\frac{dy}{dx} = 2x \ln x + x^2 \cdot \frac{1}{x}$

$= 2x \ln x + x$

$= x(2 \ln x + 1)$ $\{\ln e = 1\}$ $\{2 \ln x = \ln x^2\}$

$= x(\ln x^2 + \ln e)$

$= \underline{x(\ln e x^2)}$ ✓

10. $y = e^{t^2}$, Show that $\frac{d^2y}{dt^2} - 2t\frac{dy}{dx} - 2y = 0$

$\frac{dy}{dt} = e^{t^2} \cdot 2t = \boxed{2te^{t^2}}$

$\frac{d^2y}{dt} = 2e^{t^2} + 2t \cdot e^{t^2} \cdot 2t$ {product rule)

$\boxed{\frac{d^2y}{dt^2} = 2e^{t^2} + 4t^2e^{t^2}}$

using subtitution :

$\frac{d^2y}{dt^2} - 2t\frac{dy}{dx} - 2y$

$= \underline{\cancel{2e^{t^2}} + 4t^2e^{t^2}} - \underline{2t(2te^{t^2})} - \underline{2(\cancel{e^{t^2}})}$

$= \underline{4t^2e^{t^2} - 4t^2e^{t^2} = 0}$ ✓