Chapter 1.2 Review page 23 Maths in Action

1. (a) $\frac{x+14}{(x-4)(x+2)} = \frac{A}{(x-4)} + \frac{B}{(x+2)}$

$x + 14 = A(x+2) + B(x-4)$

let $x=4$: $4+14 = 6A \rightarrow \boxed{A=3}$

let $x=-2$: $-2+14 = -6B \rightarrow \boxed{B=-2}$

hence: $\boxed{\frac{x+14}{(x-4)(x+2)} = \frac{3}{x-4} - \frac{2}{x+2}}$

(b) $\frac{1+x-3x^2}{(x-2)(x+1)^2} = \frac{A}{(x-2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$

$1 + x - 3x^2 = A(x+1)^2 + B(x-2)(x+1) + C(x-2)$

let $x=2$: $1+2-3(2)^2 = A(2+1)^2$

$-9 = 9A \rightarrow \boxed{A=-1}$

let $x=-1$: $1-1-3(-1)^2 = C(-3) \rightarrow \boxed{C=1}$

let $x=0$ $1+0-3(0)^2 = -1(1) + (-2B) + 1(-2)$

$1 = -3 - 2B \rightarrow \boxed{B=-2}$

hence: $\boxed{\frac{1+x-3x^2}{(x-2)(x+1)^2} = \frac{-1}{x-2} - \frac{2}{x+1} + \frac{1}{(x+1)^2}}$

(c) $\frac{5x^2+6x+7}{(x-1)(x^2+2x+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+3}$

$5x^2+6x+7 = A(x^2+2x+3) + (Bx+C)(x-1)$

let $x=1$: $5(1)^2+6(1)+7 = A(1^2+2(1)+3)$

$18 = 6A \rightarrow \boxed{A=3}$

let $x=0$: $7 = 3(3) + C(-1) \rightarrow \boxed{C=2}$

let $x=-1$: $6 = 3(2) + (B(-1)+2)(-2) \rightarrow \boxed{B=2}$

hence: $\boxed{\frac{5x^2+6x+7}{(x-1)(x^2+2x+3)} = \frac{3}{x-1} + \frac{2x+2}{x^2+2x+3}}$

2. (a) $\frac{x+3}{x^3-x} = \frac{x+3}{x(x^2-1)} = \frac{x+3}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

then

$$x+3 = A(x-1)(x+1) + B(x)(x+1) + C(x)(x-1)$$

let $x=0$: $3 = A(-1)(1) \rightarrow \boxed{A=-3}$

let $x=1$: $4 = B(1)(2) \rightarrow \boxed{B=2}$

let $x=-1$: $2 = C(-1)(-2) \rightarrow \boxed{C=1}$

hence: $\boxed{\frac{x+3}{x^3-x} = \frac{-3}{x} + \frac{2}{x-1} + \frac{1}{x+1}}$

(b) $\frac{x^2-3x+3}{x^2-x^3} = \frac{x^2-3x+3}{x^2(1-x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(1-x)}$

then:

$$x^2-3x+3 = A(x)(1-x) + B(1-x) + C(x^2)$$

let $x=0$: $3 = B \rightarrow \boxed{B=3}$

let $x=1$: $1 = C \rightarrow \boxed{C=1}$

let $x=-1$: $7 = A(-1)(2) + 3(2) + 1(1) \rightarrow \boxed{A=0}$

hence: $\boxed{\frac{x^2-3x+3}{x^2-x^3} = \frac{3}{x^2} + \frac{1}{1-x}}$

* (c) $\frac{1-2x-x^2}{x^3+x^2+x} = \frac{1-2x-x^2}{x(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+1}$

CHALLENGE!

then

$$1-2x-x^2 = A(x^2+x+1) + (Bx+C)x$$

let $x=0$: $1 = A(1) \rightarrow \boxed{A=1}$

[As you can't let $x=0$ again, multiply brackets let $A=1$]

$1-2x-x^2 = x^2+x+1 + Bx^2 + Cx$ (combine like terms)

$1-2x-x^2 = (B+1)x^2 + (C+1)x + 1$ ← compare

$B+1=-1$ and $C+1=-2$ so $\boxed{B=-2}$, $\boxed{C=-3}$

hence: $\boxed{\frac{1-2x-x^2}{x^3+x^2+x} = \frac{1}{x} + \frac{-2x-3}{x^2+x+1}}$

3 (a) $\dfrac{x^2+2}{(x-1)(x+2)} = \dfrac{x^2+2}{x^2+x-2}$ divide:

$$x^2+x-2 \;\overline{)\; x^2+0x+2}$$

$$1 + \frac{-x+4}{x^2+x-2}$$

$$-(x^2+x-2)$$

$$-x+4$$

$$= 1 + \frac{-x+4}{(x-1)(x+2)}$$

$$\frac{-x+4}{(x-1)(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x+2)}$$

So $-x+4 = A(x+2) + B(x-1)$

let $x=1$: $3 = 3A \rightarrow \boxed{A=1}$

let $x=-2$: $6 = -3B \rightarrow \boxed{B=-2}$

hence: $\boxed{\dfrac{x^2+2}{(x-1)(x+2)} = 1 + \dfrac{1}{(x-1)} - \dfrac{2}{(x+2)}}$

(b) $\dfrac{x^4+2x^2-2x+1}{x^3+x}$ ← Factorable

$$x + \frac{x^2-2x+1}{x^3+x}$$

$$x^3+0x^2+x+0 \;\overline{)\; x^4+0x^3+2x^2-2x+1}$$

$$-(x^4+0x^3+x^2+0x+0)$$

$$x^2-2x+1$$

then $\dfrac{x^2-2x+1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$

$x^2-2x+1 = A(x^2+1) + (Bx+C)(x)$

let $x=0$: $1 = A \rightarrow \boxed{A=1}$

$x^2-2x+1 = Ax^2+A+Bx^2+Cx$

$1x^2-2x+1 = (A+B)x^2+Cx+A$ { expand and compares }

$A+B=1$, $A=1$ so $\boxed{B=0}$

comparing, $\boxed{C=-2}$

hence: $\dfrac{x^4+2x^2-2x+1}{x^3+x} = x + \dfrac{1}{x} + \dfrac{(-2)}{x^2+1}$

$$= \boxed{x + \frac{1}{x} - \frac{2}{x^2+1}}$$