Higher Mathematics - Practice Examination C

Please note … the format of this practice examination is different from the current format. The paper timings are different and calculators can be used throughout.

MATHEMATICS
Higher Grade - Paper I

Time allowed - 2 hours

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. Calculators may be used.
3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \\ \sin 2A &= 2\sin A \cos A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. Differentiate $\dfrac{1}{2x}\left(4\sqrt{x}+1\right)$ with respect to x. **(5)**

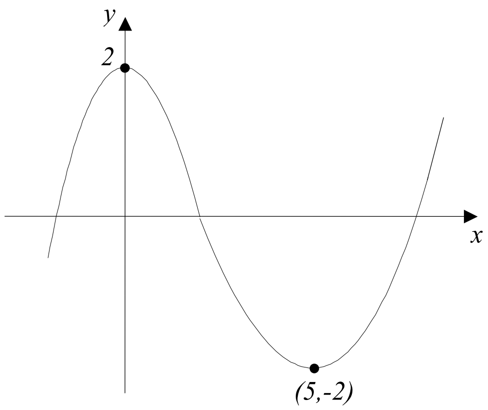
2. The perpendicular bisector of the line joining the points A(1,-2) and B(3,-4) passes through the point $(5,k)$.
Find the value of k. **(4)**

3. Three vertices of the quadrilateral $PQRS$ are $P(7,-1,5)$, $Q(5,-7,2)$ and $R(-1,-4,0)$.

(a) Given that $\overrightarrow{QP} = \overrightarrow{RS}$, establish the coordinates of S. **(2)**

(b) Hence show that SQ is perpendicular to PR. **(3)**

4. The diagram below shows the graph of $y = g(x)$.
The function has stationary points at $(0,2)$ and $(5,-2)$.

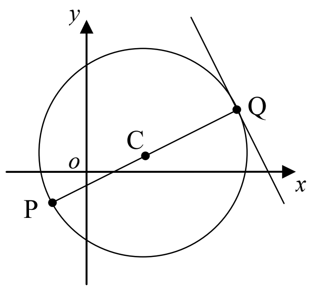


Sketch the graph of the derived function $y = g'(x)$. **(3)**

5. Solve algebraically the equation

$$\cos 2x^\circ - 6\sin x^\circ + 7 = 0 \qquad \text{for} \qquad 0 \leq x < 360.$$ **(5)**

6. A circle has as its centre C(5,1) with the point P(-4,-2) on its circumference as shown in the diagram below.

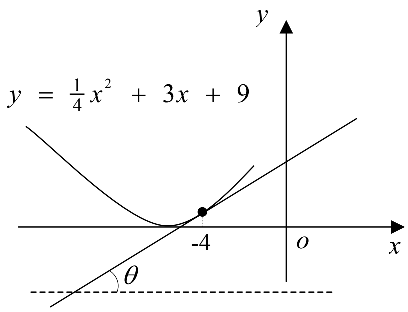


(*a*) Establish the coordinates of the point Q , given that PQ is a diameter of the circle. **(1)**

(*b*) Hence find the equation of the tangent to the circle at the point Q. **(4)**

7. What can you say about b if the equation $x + \dfrac{4}{x} = b$ has **real** roots ? **(5)**

8. Part of the graph of the curve $y = \frac{1}{4}x^2 + 3x + 9$ is shown opposite.
The diagram is not to scale.
The tangent to the curve at the point where $x = -4$ has been drawn.



Calculate the size of angle θ , the angle between the tangent and the horizontal. **(4)**

9. Two functions are defined on suitable domains and are given as

$$f(x) = x + 3 \qquad \textit{and} \qquad g(x) = x^2 - 1 .$$

(*a*) Find an expression for the function h when $h(x) = g\big(f(x)\big)$. **(2)**

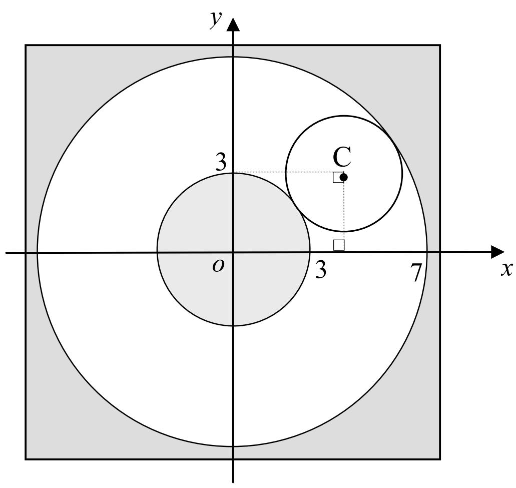
(*b*) Find the value(s) of a given that $h(a) = f(a) + 1$ **(3)**

10. Evaluate $\displaystyle\int_{-1}^{1} \left(2 + \frac{1}{x^2}\right)^2 \, dx$ **(5)**

11. Find the value of c if $x+1$ is a factor of the expression

$$cx^3 + (1-c)x^2 - (2c+1)x + c$$ **(4)**

12. The diagram below shows part of a watch mechanism.
The small circular disc, centre C, is free to rotate around the centre circle of radius 3 units.
It is held in its groove by the external circle which has a radius of 7 units.
Both the internal and external circles are centered on the origin.

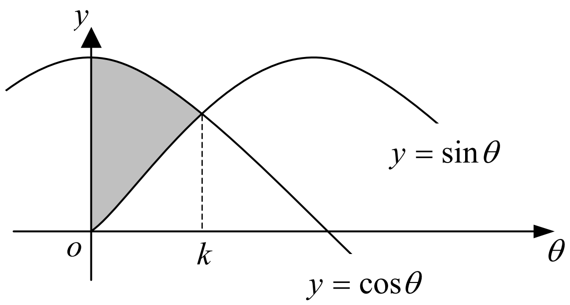


With the mechanism **fixed** in the position shown, establish the equation of the small circular disc centre C. **(5)**

13. By taking logarithms in the base two of both sides of the following equation, find algebraically the value of x

$$8^{x+1} = 4^{2x-3}$$ **(4)**

14. The sketch below shows part of the graphs of $y = \sin\theta$ and $y = \cos\theta$.



(a) Write down the value of k in radians. **(1)**

(b) Hence show that the exact area of the shaded region is $\sqrt{2} - 1$ square units. **(5)**

15. A sequence is defined by the recurrence relation $U_{n+1} = aU_n + b$, where a and b are constants.

(a) Given that $U_0 = 4$ and $b = -8$, express U_2 in terms of a. **(2)**

(b) Hence find the value of a when $U_2 = 88$ and $a > 0$. **(3)**

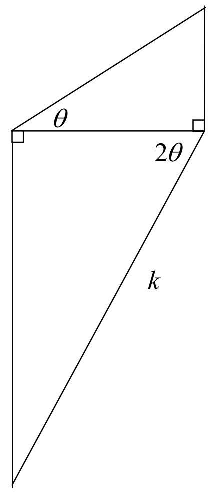
(c) Given that $S_3 = U_1 + U_2 + U_3$, calculate the value of S_3. **(2)**

16. A function is defined as $g(\theta) = 2\sin 2\theta - 4\cos^2\theta$.

Show that $g'(\theta)$ can be written in the form

$$g'(\theta) = 4(\cos 2\theta + \sin 2\theta) \qquad \textbf{(4)}$$

17. Consider the diagram opposite where both θ and 2θ are acute.



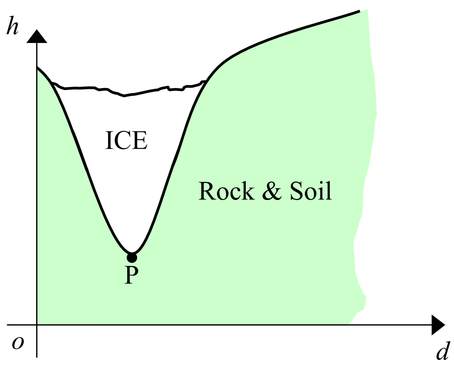
(*a*) Given that $\tan\theta = \frac{1}{\sqrt{2}}$, find the **exact** value of

i) $\cos\theta$

ii) $\cos 2\theta$. **(4)**

(*b*) Hence find the **exact** value of k . **(2)**

18. The graph below shows the cross section of a small glacier.
The horizontal axis indicates the amount of level drift, d metres, and has a scale of 1 unit represents 150 metres.
The vertical axis is the approximate height, h metres, above sea level and has a scale of 1 unit represents 100 metres.



(*a*) The curved lower edge of the glacier is found to be the function defined as

$$h(d) = \left[\frac{-4}{d^2 - 4d + 5} \right] + 6 \quad , \quad \text{for} \quad 0 \le d \le 5 .$$

Express the function in the form

$$h(d) = \left[\frac{-4}{(d-a)^2 + b} \right] + 6$$ **(2)**

(*b*) Hence state the minimum value of h and the corresponding value of d . **(2)**

(*c*) With reference to the origin, and using the scales given, state the position of P in metres. **(1)**

[END OF QUESTION PAPER]

1. For expanding brackets and dealing with root sign
(1 - for bringing x up, 1 - for root sign, 1 - for expanding)

i.e $\dfrac{x^{-1}}{2}\left(4x^{\frac{1}{2}} + 1\right) = 2x^{-\frac{1}{2}} + \frac{1}{2}x^{-1}$ (or equivalent) .... (3)

then $\dfrac{d}{dx} = -x^{-\frac{3}{2}} - \frac{1}{2}x^{-2}$ (or equivalent) ... 1 each term $\Rightarrow$ .... (2)

[5 marks]

2. For $m_{AB} = -1 \quad \therefore \quad m_{per.} = 1$ .... (1)
For mid-point of AB .... M(2,-3) .... (1)
For point + gradient in $y - b = m(x - a)$ or equiv.
to ans. $y = x - 5$ .... (1)
For sub. in $(5,k)$ to answer $k = 0$ .... (1)

[4 marks]

3. (a) For $\underset{\sim}{p} - \underset{\sim}{q} = \underset{\sim}{s} - \underset{\sim}{r}$ (or equiv.) .... (1)

For answer $\underset{\sim}{s} = \underset{\sim}{p} - \underset{\sim}{q} + \underset{\sim}{r} = \begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ -7 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\therefore$ S(1, 2, 3) .... (1)
(*no marks off for leaving as a column vector*) **[2 marks]**

(b) For $SQ \bullet PR = 0$ if perpendicular (stated or implied) .... (1)

then $SQ = \begin{pmatrix} 4 \\ -9 \\ -1 \end{pmatrix}$ and $PR = \begin{pmatrix} -8 \\ -3 \\ -5 \end{pmatrix}$ .... (1)

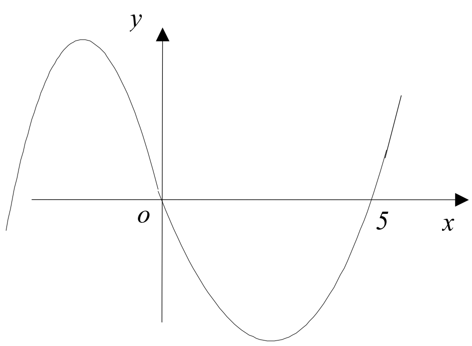
for $SQ \bullet PR = \begin{pmatrix} 4 \\ -9 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} -8 \\ -3 \\ -5 \end{pmatrix} = -32 + 27 + 5 = 0$ .... (1)

[3 marks]

4.
For parabolic in shape .... (1)

For correct roots .... (1)

For annotation .... (1)



[3 marks]

5. For using correct replacement i.e $1 - 2\sin^2 x^\circ$ .... (1)

For subst. and simplifying to $-2(\sin^2 x^\circ + 3\sin x^\circ - 4) = 0$ (or equiv.) .... (1)

For factorising and solving to ... $\sin x^\circ = -4$ *or* $\sin x^\circ = 1$ .... (1)

For discarding $\sin x^\circ = -4$ as having no solution .... (1)

For final answer .... $x = 90^\circ$ .... (1)

[5 marks]

6. (a) For stepping out (or equiv) to Q(14,4) .... (1)

[1 mark]

(b) For attempting or calculating m of radius - $m_r = \frac{1}{3}$ .... (1)

($\frac{3}{9}$ is fine)

For knowing and using perpendicular gradient , $m_t =$ -3 .... (1)

For knowing to use $y - b = m(x - a)$ (or equiv.) .... (1)

For correct point and gradient to $y - 4 = -3(x - 14)$ .... (1)

Final answer could be in the form $3x + y = 46$ (or equiv.) **[4 marks]**

7. For turning equ. into standard quad. form ... $x^2 - bx + 4 = 0$ .... (1)

For knowing that for real roots $b^2 - 4ac \geq 0$ (stated or implied) .... (1)

For $a = 1$, $b = -b$ *and* $c = 4$ .... (1)

For evaluating discrim. to $b^2 - 16 \geq 0$ (or equiv.) .... (1)

For final answer that b cannot lie between -4 *and* 4 (or equiv.) .... (1)

[5 marks]

8. For strategy ...... differentiating .... (1)

For differentiating to $\frac{dy}{dx} = \frac{1}{2}x + 3$ and evaluating @ $x = -4$ .... (1)

to value $\frac{dy}{dx} = 1$ = the gradient of the tangent (m) .... (1)

For $\tan\theta = m$ $\therefore \tan\theta = 1$ then $\theta = 45^\circ$ .... (1)

[4 marks]

9. (a) For $g(f(x)) = (x+3)^2 - 1$ .... (1)

then ans. $h(x) = x^2 + 6x + 8$ .... (1)

[2 marks]

(b) For realising $a^2 + 6a + 8 = (a + 3) + 1$ (or equivalent) .... (1)

For $a^2 + 5a + 4 = 0$ .... (1)

For solving to ans. $a = -4$ *or* $a = -1$ .... (1)

[3 marks]

10. For expanding to $\int_{-1}^{1} (4 + 4x^{-2} + x^{-4})\ dx$ .... (1)

For integrating to $\left[4x + \dfrac{4x^{-1}}{-1} + \dfrac{x^{-3}}{-3} \right]_{-1}^{1}$ .... (1)

For simplifying to $\left[4x - \dfrac{4}{x} - \dfrac{1}{3x^3} \right]_{-1}^{1}$ .... (1)

For knowing to subst. and <u>subtract</u> .... (1)

For answer $\frac{1}{3} + (-\frac{1}{3}) = -\frac{2}{3}$ .... (1)

[5 marks]

11. For deciding on method (synth. div.) and using -1 as the dividend .... (1)

For setting up to ...... $-1 \quad c \quad (1-c) \quad -(2c+1) \quad c$ .... (1)

For correct quotient ... $c \quad 1-2c \quad -2 \quad c+2$ .... (1)

For knowing to solve to zero $c+2=0 \quad \therefore \quad c=-2$ .... (1)

[4 marks]

12. Although little direction given, pupils should be able to gain marks by trying to establish a centre and a radius. A wrong centre etc. still produces full marks for the circle techniques.

For realising $d = 4$ and therefore $r = 2$ .... (1)

For now attempting to locate a centre and realising it is $(k, 3)$ (or equivalent) .... (1)

For using Pyth. or other to find $x = 4 \quad \therefore$ C(4,3) .... (1)

For using $(x-a)^2 + (y-b)^2 = r^2$ and substituting <u>any</u> centre and radius in correctly ... . (2)

Answer : $(x-4)^2 + (y-3)^2 = 4 \quad or \quad x^2 + y^2 - 8x - 6y + 21 = 0$

[5 marks]

13. For taking the logs ... $\log_2 8^{x+1} = \log_2 4^{2x-3}$ .... (1)

For realeasing the powers to $(x+1)\log_2 8 = (2x-3)\log_2 4$ .... (1)

For evaluating the logs to $(x+1).3 = (2x-3).2$ .... (1)

For solving equation to ans. $x = 9$ .... (1)

(if pupils solve by index laws alone etc. $\frac{3}{4}$ marks) **[4 marks]**

14. (a) For ans. ..... $k = \frac{\pi}{4}$ .... (1)

[1 marks]

(b) For knowing $A = \int_0^{\frac{\pi}{4}} (\cos\theta - \sin\theta)\, dx$ (or sep. integ. subtr.) .... (1)

For .... $= [\sin\theta + \cos\theta]_0^{\frac{\pi}{4}}$ .... (1)

For sub. to $= (\sin\frac{\pi}{4} + \cos\frac{\pi}{4}) - (\sin 0 + \cos 0)$ .... (1)

For answer $A = \dfrac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1\ sq.units$ .... (2)

[5 marks]

15. (a) For $U_1 = a(4) - 8 = 4a - 8$ .... (1)

For $U_2 = a(4a-8) - 8 = 4a^2 - 8a - 8$ .... (1)

[2 marks]

(b) For $4a^2 - 8a - 8 = 88$ .... (1)

For equating to zero and factorising to $4(a-6)(a+4) = 0$ .... (1)

For correct ans. (discarding a = -4) $\therefore \; a = 6$ .... (1)

[3 marks]

(c) For evaluating $U_1 = 16$ *and* $U_3 = 520$ (1 each) .... (1)

For $S_3 = 16 + 88 + 520 = 624$ .... (1)

[2 marks]

16. 1 mark for each derivative i.e. For $4\cos 2\theta$ .... (1)

For $-8\sin\theta\cos\theta$ .... (1)

Giving answer $g'(\theta) = 4\cos 2\theta - (-8\sin\theta\cos\theta)$

For tidying up to $g'(\theta) = 4(\cos 2\theta + 2\sin\theta\cos\theta)$ ... (or equiv.) .... (1)

For final replacement to ans. $g'(\theta) = 4\left(\cos 2\theta + \sin 2\theta\right)$ .... (1)

[4 marks]

17. (a) i) For hypotenuse $h = \sqrt{3}$ .... (1)

then $\cos\theta = \dfrac{\sqrt{2}}{\sqrt{3}}$ (or equiv.) .... (1)

ii) For $\cos 2\theta = 2\cos^2\theta - 1$ (electing to use) .... (1)

for subst. for $\cos\theta$ then ans. - $\cos 2\theta = \frac{1}{3}$ .... (1)

[4 marks]

(b) For equating $\cos 2\theta = \dfrac{1}{3} = \dfrac{\sqrt{2}}{k}$ (or equivalent) .... (1)

For answer - $k = 3\sqrt{2}$ .... (1)

[2 marks]

18. (a) For ..... $\left[\,(d-2)^2 - 4\right] + 5$ .... (1)

For ans. $h(d) = \left[\dfrac{-4}{(d-2)^2 + 1}\right] + 6$ .... (1)

[2 marks]

(b) For $h_{\min} = 2$ @ $d = 2$ (1 each) .... (2)

[2 marks]

(c) For answer P(300,200) (or equiv., words etc.) .... (1)

[1 mark]

Total : 87 marks

Higher Mathematics - Practice Examination C

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MATHEMATICS

Higher Grade - Paper II

Time allowed - 2 hours 30 mins

Read Carefully

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FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned}
\sin\left(A \pm B\right) &= \sin A \cos B \pm \cos A \sin B \\
\cos\left(A \pm B\right) &= \cos A \cos B \mp \sin A \sin B \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A \\
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\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

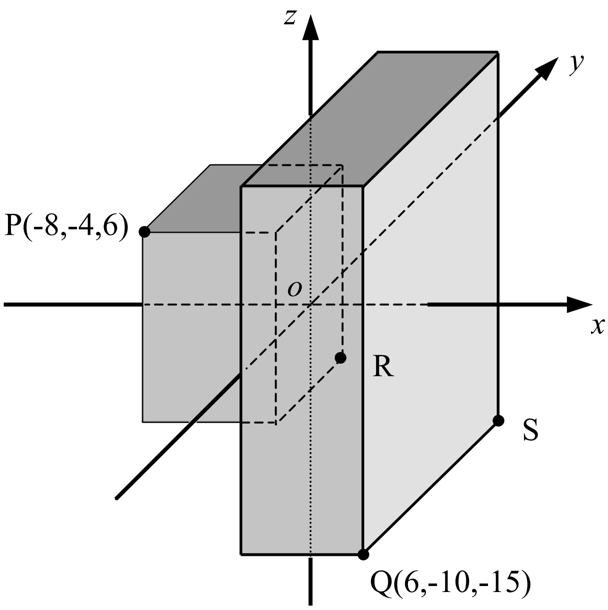
Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. A metal casting is in the shape of two cuboids.

The casting is positioned relative to a set of rectangular axes as shown below.
Both cuboids have been centred along the x-axis.



(*a*) Given that corners P and Q have coordinates $(-8,-4,6)$ and $(6,-10,-15)$ respectively , write down the coordinates of corners R and S. **(2)**

(*b*) Hence show that the corners P , R and S are collinear. **(2)**

(*c*) Find $\overrightarrow{RP}$ and $\overrightarrow{RQ}$ **(1)**

(*d*) Calculate the size of $\angle PRQ$. **(5)**

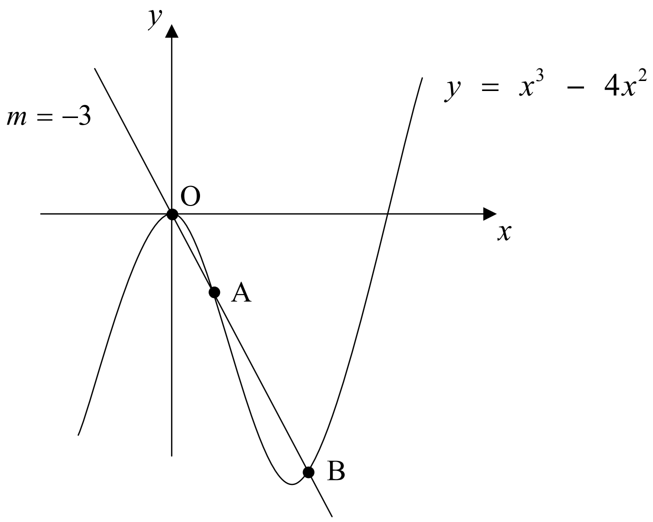
2. Two functions are defined on suitable domains as

$$f(x) = \frac{3x+p}{2} \qquad \text{and} \qquad g(x) = p^2 + x \quad , \text{ where } p \text{ is a constant.}$$

(*a*) Find expressions for $f(g(x))$ and $g(f(x))$. **(3)**

(*b*) Find the value of the constant p when $f(g(x)) - g(f(x)) = 18$ and $p > 0$. **(4)**

3. The line which passes through the origin with gradient -3 intersects the curve with equation $y = x^3 - 4x^2$ at two further points A and B , as shown in the diagram below.



(a) Establish the coordinates of A and B. **(5)**

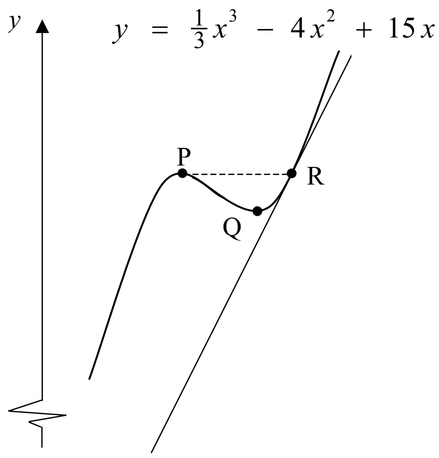
(b) Hence show that OA is half the length of AB. **(3)**

4. A curve has as its equation $y = \frac{1}{3}x^3 - 4x^2 + 15x$.

Part of the graph of this curve is shown in the diagram opposite.

The diagram is not drawn to scale.

The tangent at the point R on the curve is also shown



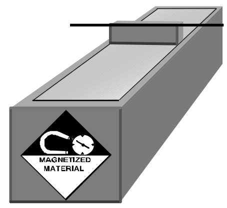
(a) Find the coordinates of the stationary points. **(6)**

(b) Establish the coordinates of the point R given that PR is parallel to the x – axis and that the x – coordinate of R is a **whole** number. Hence find the equation of the tangent at R. **(5)**

(c) This tangent meets the curve at a second point, state the coordinates of this second point. **(1)**

5. The diagram opposite shows a small bar magnet which is part of an electrical control circuit.

When first placed in the circuit the magnetic strength of the magnet is rated at 100 *mfu* (magnetic flux units).



(*a*) When the circuit is switched on, heat is produced.
This heat disturbs the dipoles within the domains of the magnet producing a demagnetization of the magnet (i.e. a decrease in magnetic strength).

During any six hour period, when the circuit is running, the magnet is known to lose 4% of the magnetic strength it had at the beginning of the period.

Calculate the magnetic strength of the magnet after the circuit has been running continuously for 24 hours. **(3)**

(*b*) At the end of each 24 hour period, the circuit (with the magnet in place) is automatically passed through a very intense electric field. This allows the magnet to regain some of its lost strength.
A single pass through the field and the magnet regains 12 *mfu* of strength.

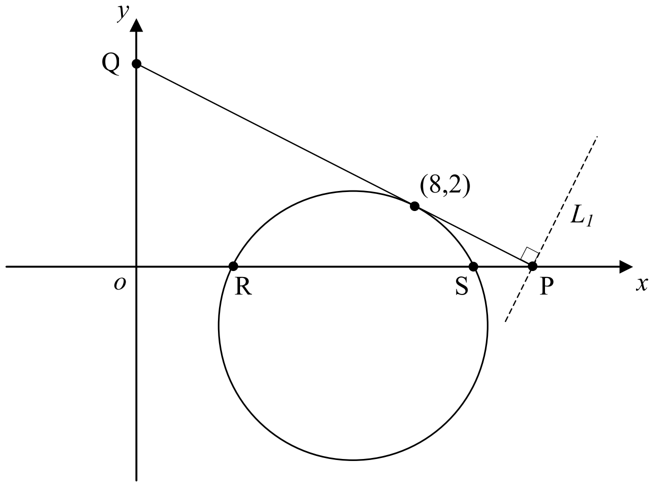
The 24 hour cycle described above is now left to run uninterrupted for a number of weeks.

Given that a magnet which falls below a strength of 77 *mfu* will not function properly in the circuit, comment on whether or not the above conditions are satisfactory ? **(4)**
Your answer must be accompanied with the appropriate working.

(*c*) Although an expensive option the company decide to change their strategy.
They buy new equipment so that at the end of each 24 hour period the circuit is passed through an electric field many times stronger than the original.
The result for the magnet is a gradual increase in magnetic strength until "magnetic saturation" is reached.
(It is known that the strength of a magnet cannot be increased beyond a certain limit.)

Calculate the strength of this particular magnet when "magnetic saturation" occurs given that the new equipment is now adding 16 *mfu* at the end of each 24 hour period. **(3)**

6. The diagram below shows the circle with equation $x^2 + y^2 - 12x + 4y + 20 = 0$.
The tangent at the point (8,2) on the circle meets the x – axis at P and the y – axis at Q as shown.
The circle cuts the x – axis at the points R and S .



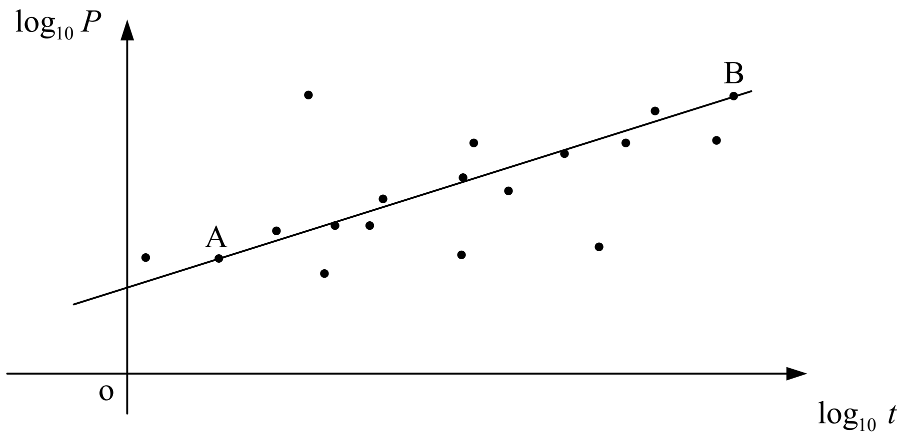
(*a*) Find the coordinates of the points P and Q. **(6)**

(*b*) Given that the perpendicular to the line PQ through P (line L_1) is also a tangent to the circle, show that its point of tangency, T, is (10,-4). **(3)**

(*c*) Prove that the line TQ passes through the mid-point of the chord RS. **(3)**

7. In a scientific experiment, corresponding readings of pressure (P) and temperature (t) were taken and recorded.

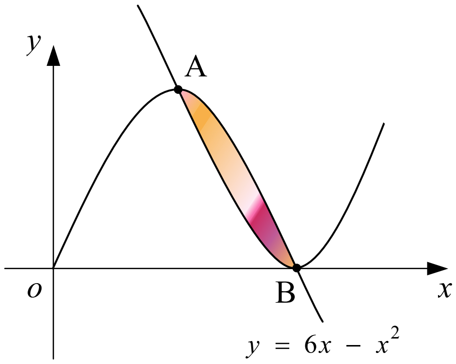
The data was put into logarithmic form and the graph of $\log_{10} P$ against $\log_{10} t$ was plotted.
This resulting graph, with the line of best fit added, can be seen below.



(*a*) Explain why the original data takes the form $P = kt^n$, where k and n are constant integers. **(4)**

(*b*) Given that the points A and B on the graph have coordinates $(1\cdot6, 11\cdot98)$ and $(1\cdot844, 13\cdot45)$ respectively, establish the equation of the line and state the values of n and k . **(4)**

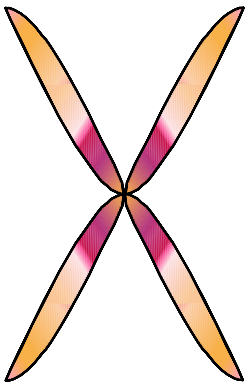
8. The diagram below shows parts of the graphs of two quadratic curves.



(a) The curve with its maximum turning point at A has as its equation $y = 6x - x^2$.
Establish the coordinates of A and B . **(2)**

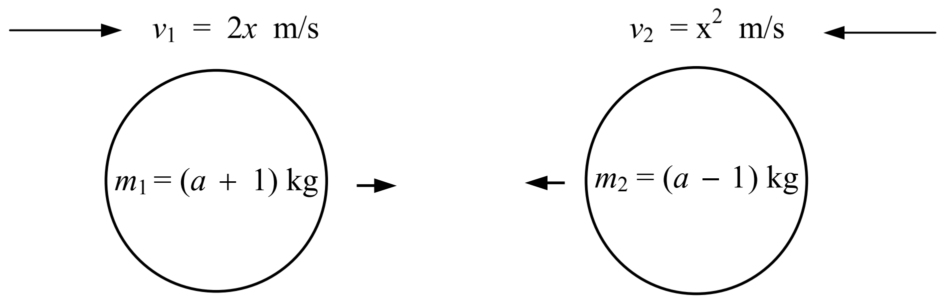
(b) The second quadratic curve passes through the point A and has its turning point at B.
Show that this second curve has as its equation $y = x^2 - 12x + 36$. **(3)**

(c) The design below has been created by reflecting the area enclosed between the two curves in the horizontal and vertical axes.
Calculate the area of this design in square units.



(6)

9. The two objects shown below collide on a horizontal surface.
The momentum (M) of each object before the collision can be found by using the formula $M = mv$, where m is the mass of the object, in *kilograms*, and v is its velocity in *metres per second*.



(*a*) Write down expressions, in terms of a and x , for the two momentums M_1 and M_2 before the collision. **(1)**

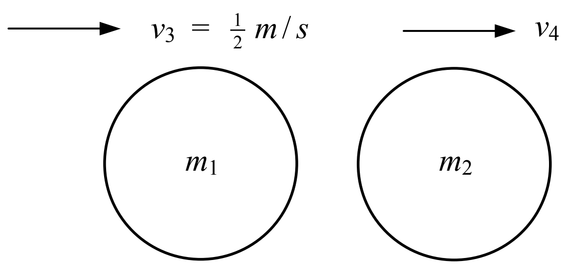
(*b*) Given that the difference between the two momentums before the impact is 9 , with the object on the left having the greater momentum, show that the following equation can be constructed

$$(a - 1)x^2 - (2a + 2)x + 9 = 0$$ **(2)**

(*c*) Given that $a > 2$, find the value of a for which the equation $(a - 1)x^2 - (2a + 2)x + 9 = 0$ has **equal** roots and hence write down the mass of each object. **(5)**

(*d*) Calculate the velocity of each object **before** impact. **(2)**

(*e*) By conservation of momentum the combined momentum after the collision is equal to the difference in momentum before the collision. This produces the formula $m_1 v_3 + m_2 v_4 = 9$ for after the impact.
The diagram below shows the situation immediately after impact.



Using the above diagram and the given formula, calculate the speed of the lighter object after the impact. **(2)**

[END OF QUESTION PAPER]

Higher Mathematics Practice Exam C Marking Scheme - Paper 2

1. (a) 1 each for R(0,4,-6) and S(6,10,-15) .... (2)

[2 marks]

(b) For two disp. e.g. $\vec{PR} = \begin{pmatrix} 8 \\ 8 \\ -12 \end{pmatrix}$ *and* $\vec{RS} = \begin{pmatrix} 6 \\ 6 \\ -9 \end{pmatrix}$ .... (1)

For stating since $\vec{PR} = \frac{4}{3}\vec{RS}$, then *P*, *R* ans *S* are collinear (or equiv) ...... (1)

[2 marks]

(c) For disp. $\vec{RP} = \begin{pmatrix} -8 \\ -8 \\ 12 \end{pmatrix}$ *and* $\vec{RQ} = \begin{pmatrix} 6 \\ -14 \\ -9 \end{pmatrix}$ .... (1)

(basically only finding $\vec{RQ}$, since $\vec{RP}$ is just a reversal) **[1 mark]**

(d) For knowing to use scalar product and that $\cos\theta = \dfrac{\vec{RP}.\vec{RQ}}{|RP|.|RQ|}$ .... (1)

For scalar product $\vec{RP}.\vec{RQ} = -44$ .... (1)

For both magnitudes $\sqrt{272}$ *and* $\sqrt{313}$ .... (1)

For $\cos\theta = \dfrac{-44}{\sqrt{85136}}$ (or equiv.) .... (1)

For ans. angle PRQ = $98\cdot 7^{\circ}$ .... (1)

[5 marks]

2. (a) For knowing the technique .... (1)

For ... $f(g(x)) = \dfrac{3(p^2 + x) + p}{2}$, simpl.^{d} or in any form .... (1)

For ... $g(f(x)) = p^2 + \dfrac{3x + p}{2}$, simpl.^{d} or in any form .... (1)

[3 marks]

(b) For $\dfrac{3(p^2 + x) + p}{2}$ - $\left[p^2 + \dfrac{3x+p}{2} \right]$ = 18 , simpl.^{d} ,any form .... (1)

For simplifying correctly etc to ans. $p = 6$ (many forms) .... (3)

[4 marks]

3. (a) For deciding on the strategy of a system of equ. .... (1)

For the equation of the line $y = -3x$ .... (1)

For equating to .... $-3x = x^3 - 4x^2$ .... (1)

For solving to $x(x-1)(x-3) = 0 \;\therefore\; x = 1$ *or* $x = 3$ .... (1)

For final answers A(1,-3) and B(3,-9) .... (1)

[5 marks]

(b) For strategy i.e. pyth. (vector magnitude, etc.) .... (1)

For the two lengths i.e. OA = $\sqrt{10}$ and AB = $2\sqrt{10}$ .... (1)

For final statement and keeping to exact values .... (1)

(if pupil approximates the lengths ..... 2 out of 3 marks) **[3 marks]**

4. (a) For attempting to differentiate .... (1)

For $\frac{dy}{dx} = 0$ (stated or implied) .... (1)

For diff.^{g} and solving to $x = 3$ or $x = 5$ .... (2)

1 mark for each y – coordinate ... P(3,18) and Q(5,16$\frac{2}{3}$) .... (2)

[6 marks]

(b) For realising y – coordinate of R is 18 (stat. or imp.) .... (1)

For trial and error realising that $x > 5$ until $x = 6$ .... (1)

(some pupils may use synthetic division)

For $\frac{dy}{dx} = m$ .... (1)

For evaluating derivative for $x = 6$ to ... $m = 3$ .... (1)

For point and gradient to answer (any form) ... $y = 3x$ .... (1)

[5 marks]

(c) By inspection both pass through the origin $\therefore$ second point is (0,0) .... (1)

[1 mark]

5. (a) For correct multiplier i.e. $0 \cdot 96$ .... (1)

For realising one calculation ... $= (0 \cdot 96)^4 \times 100$ .... (1)

then $= 84 \cdot 935$ .... (1)

(rounding not a problem) **[3 marks]**

(b) For setting up recurr. .. $U_{n+1} = (0 \cdot 96)^4 . U_n + 12$ (or equiv.) .... (1)

For setting out at least five lines of calculations .... (1)

For knowing to look at **lower value** before adding 12 .... (1)

For discovering ans.... that by end of day 5 lower value

has arrived at $\approx 76 \cdot 64$ $\therefore$ strength < 77 and not acceptable. .... (1)

(for only looking at upper value and $\therefore$ o.k. ... fundamental error, $\frac{2}{4}$) **[4 marks]**

(c) For knowing "limit" formula or a limit is mentioned. .... (1)

For formula ..... $L = \frac{16}{1 - (0 \cdot 96)^4}$ (or equiv.) .... (1)

For answer ..... Magnetic saturation at $106 \cdot 2$ *mfu* .... (1)

(for setting up another sequence etc. $\frac{2}{3}$) **[3 marks]**

6. (a) For drawing out centre C(6,-2) .... (1)

For establishing $m_{tan} = -\frac{1}{2}$ .... (2)

For using a point and a gradient in $y - b = m(x - a)$ .... (1)

Equation of tangent - $x + 2y = 12$ (or equiv.) .... (1)

For points P(12,0) and Q(0,6) .... (1)

[6 marks]

(b) For realising a rectangle is present .... (1)

For correct x and y to ans. - T(10,-4) .... (2)

[3 marks]

(c) Mid-pt. of chord (vertically above centre) - M(6,0) .... (1)

(pupils may miss this and solve circle equ. for $y = 0$)

For proving (6,0) lies on the line TQ (various methods) .... (2)

[3 marks]

7.	(a)	For .... since log graph is a straight line then it takes the form $Y = mx + c$ (stated or implied)	....	(1)
		For $\log_{10} P = n\log_{1o} t + \log_{10} k$	....	(1)
		then $\log_{10} P = \log_{10} kt^n$	....	(1)
		then logs away to .... $P = kt^n$	....	(1)
				[4 marks]
	(b)	For knowing that $m = n$ and calculating it to $n = 6$ (integer)	....	(1)
		For sub. a point + n into $\log_{10} P = n\log_{1o} t + \log_{10} k$	....	(1)
		For simplifying down to $\log_{10} k \approx 2\cdot 38$	....	(1)
		For ans. for k ..... $k = 240$	....	(1)
		(No marks deducted for rounding or inaccuracy)		**[4 marks]**
8.	(a)	For ans. A(3,9) and B(6,0) ... 1 each	....	(2)
				[2 marks]
	(b)	For establ. a perfect square or being aware of correct form	....	(1)
		For arriving at ans $y = x^2 - 12x + 36$ (or equiv.)	....	(1)
		For checking that the point (3,9) lies on curve (i.e. no multiplier)	....	(1)
				[3 marks]
	(c)	For setting up integral $A = \int_3^6 \left(18x - 2x^2 - 36\right)$ (or equiv)	....	(2)
		For integrating to $A = \left[9x^2 - \frac{2x^3}{3} - 36x\right]_3^6$	....	(1)
		For substituting and knowing to subtract	....	(1)
		For calculating ans. $A = (-36) - (-45) = 9$ square units	....	(1)
		For final area of design $= 4 \times 9 = 36$ square units.	....	(1)
				[6 marks]
9.	(a)	$M_1 = 2x(a+1)$, $M_2 = x^2(a-1)$ in any form	....	(1)
				[1 mark]
	(b)	For equating to 9 (stated or implied) i.e. $2x(a+1) - x^2(a-1) = 9$	....	(1)
		For simplification to $(a - 1)x^2 - (2a + 2)x + 9 = 0$	....	(1)
				[2 marks]
	(c)	For stating "for equal roots - $b^2 - 4ac = 0$"	....	(1)
		For $a = (a-1)$, $b = -(2a+2)$ and $c = 9$	....	(1)
		For sub. then to $4(a^2 - 7a + 10) = 0$	....	(1)
		For solving and discarding $a = 2$ then ans. $\underline{a = 5}$	....	(1)
		From a the mass of each object .. $M_1 = 6$ kg , $M_2 = 4$ kg	....	(1)
				[5 marks]
	(d)	For sub. a in equation to give $4x^2 - 12x + 9 = 0$	....	(1)
		For solving to answer $x = \frac{3}{2}$	....	(1)
				[2 marks]
	(e)	For setting up equ. $\frac{1}{2}(6) + v_4(4) = 9$ in any form	....	(1)
		For solving to answer - velocity $v_4 = 1\frac{1}{2}$ m/s	....	(1)
				[2 marks]

Total : 98 marks