

1. (a) 
$$\begin{array}{r} 2x + y = 1 \xrightarrow{\times 2} 4x + 2y = 2 \\ 3x - 2y = 5 \rightarrow \underline{3x - 2y = 5} \\ \hline 7x = 7 \\ \boxed{x = 1} \end{array}$$
 
$$\begin{array}{r} 2(1) + y = 1 \\ \underline{y = -1} \end{array}$$

(b) 
$$\left( \begin{array}{cc|c} 2 & 1 & 1 \\ 3 & -2 & 5 \end{array} \right) \rightarrow \left( \begin{array}{cc|c} 2 & 1 & 1 \\ 7 & 0 & 7 \end{array} \right)$$
 
$$\begin{array}{r} 7x = 7 \\ \boxed{x = 1} \end{array}$$
 
$$\begin{array}{r} 2(1) + y = 1 \\ \underline{y = -1} \end{array}$$

2. (i) 
$$\begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 0 \end{pmatrix}$$

ii 
$$\left( \begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 3 & 2 & -1 & 6 \\ 1 & -1 & 0 & 0 \end{array} \right) \xrightarrow{\text{rearrange}} \left( \begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 2 \\ 3 & 2 & -1 & 6 \end{array} \right)$$

$$\left( \begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 3 & 1 & 2 \\ 0 & 5 & -1 & 6 \end{array} \right)$$
 
$$\begin{array}{l} \times 5 \\ \times 3 \end{array}$$
 
$$\left( \begin{array}{ccc|c} 0 & 15 & 5 & 10 \\ 0 & -15 & 3 & -18 \end{array} \right)$$

(b) 
$$\left( \begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & 8 & -8 \end{array} \right)$$
 (c) 
$$\begin{array}{r} 8z = -8 \\ \boxed{z = -1} \end{array}$$
 
$$\begin{array}{r} 3y - 1 = 2 \\ 3y = 3 \\ \boxed{y = 1} \end{array}$$
 
$$\begin{array}{r} x - 1 + 0 = 0 \\ \boxed{x = 1} \end{array}$$
 
$$\boxed{(1, 1, -1)}$$

3. 
$$\left( \begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -1 & -1 & 14 \\ 1 & -2 & 1 & 4 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -3 & 12 \\ 0 & -3 & 0 & 3 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -3 & 12 \\ 0 & 0 & 3 & -9 \end{array} \right)$$

$$\begin{array}{r} 3z = -9 \\ \boxed{z = -3} \end{array}$$
 
$$\begin{array}{r} -3y - 3(-3) = 12 \\ -3y = 3 \\ \boxed{y = -1} \end{array}$$
 
$$\begin{array}{r} x - 1 - 3 = 1 \\ \boxed{x = 5} \end{array}$$

$$\boxed{(5, -1, -3)}$$



4.  $x, y$   
 $(3, 1) \quad a x^2 + b y^2 + c x - 8 y + 1 = 0$   
 $a(3)^2 + b(1)^2 + c(3) - 8(1) + 1 = 0$

②  $9a + b + 3c = 7$

$x, y$   
 $(-1, 1) \quad a(-1)^2 + b(1)^2 + c(-1) - 8(1) + 1 = 0$

①  $a + b - c = 7$

$x, y$   
 $(1, 2) \quad a(1)^2 + b(2)^2 + c(1) - 8(2) + 1 = 0$

③  $a + 4b + c = 15$

{ choose best arrangement }

$$\left( \begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 9 & 1 & 3 & 7 \\ 1 & 4 & 1 & 15 \end{array} \right) \quad \left( \begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & -8 & 12 & -56 \\ 0 & 3 & 2 & 8 \end{array} \right) \begin{array}{l} \times 3 \\ \times 8 \end{array} \left( \begin{array}{ccc|c} 0 & -24 & 36 & -168 \\ 0 & 24 & 16 & 64 \end{array} \right)$$

$$\left( \begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & -8 & 12 & -56 \\ 0 & 0 & 52 & -104 \end{array} \right) \quad \left( \begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & -2 & 3 & -14 \\ 0 & 0 & 1 & -2 \end{array} \right) \quad \boxed{c = -2}$$

$-2b - 6 = -14$   
 $-2b = -8$   
 $\boxed{b = 4}$

~~(a)  $x^2 + 4y^2 - 2x - 8y + 1 = 0$~~

(b)  $a + 4 + 2 = 7$   
 $\boxed{a = 1}$

(c)  $x^2 + 4y^2 - 2x - 8y + 1 = 0$

5.  $\left( \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 1 & 0 & 5 \\ 0 & 1 & 1 & -2 \end{array} \right) \quad \left( \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 1 & -2 \end{array} \right) \xrightarrow{\text{sw A, B}} \left( \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & -1 & 3 \end{array} \right)$

$$\left( \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -3 \end{array} \right) \quad \left( \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array} \right) \quad \boxed{\begin{array}{l} x = 4 \\ y = 1 \\ z = -3 \end{array}}$$



6. 
$$\left( \begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 3 & 1 & p & q \end{array} \right) \xrightarrow{\text{SWAP}} \left( \begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 1 \\ 3 & 1 & p & q \end{array} \right) \left( \begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -3 & -3 & -1 \\ 0 & -5 & (p-6) & (q-3) \end{array} \right)$$

$$\left( \begin{array}{ccc|c} 0 & 15 & 15 & 3 \\ 0 & -15 & (3p-18) & (3q-9) \\ 0 & 0 & 3p-3 & 3q-4 \end{array} \right)$$

$$\left( \begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -3 & -3 & -1 \\ 0 & 0 & (3p-3) & (3q-4) \end{array} \right)$$

a) no solution when  $3p-3=0$   
 $3p=3$  and  $q \neq \frac{4}{3}$   
 $p=1$  — no solution

b) infinite solutions when  $p=1$  and  $3q-4=0$   
 $3q=4$   
 $q=\frac{4}{3}$

when  $p=1$  and  $q=\frac{4}{3}$