After Summer Holiday Revision Weeks 1&2

Solutions

Differentiate Q1 - Q9

① $\frac{dy}{dx}(2x^4+1)^6$ use chain rule then simplify

$= 6(2x^4+1)^5(8x^3) = \boxed{48x^3(2x^4+1)^5}$

② $\frac{dy}{dx}\frac{1}{\sqrt{x^2-5x}} = \frac{1}{(x^2-5x)^{\frac{1}{2}}} = (x^2-5x)^{-\frac{1}{2}}$ {differentiable FORM}

use chain rule:

$\frac{dy}{dx}(x^2-5x)^{-\frac{1}{2}} = -\frac{1}{2}(x^2-5x)^{-\frac{3}{2}}(2x-5)$

$= \boxed{-\frac{(2x-5)}{2\sqrt{(x^2-5x)^3}}}$ {simplified}

③ $x^4(2x-7)^5$ use product rule.

$\frac{dy}{dx} = 4x^3(2x-7)^5 + x^4\cdot 5(2x-7)^4\cdot 2$ {simplify}

$= \boxed{4x^3(2x-7)^5 + 10x^4(2x-7)^4}$ no need to go further on exam!

factorise to get "book" answer

$\underbrace{2x^3(2x-7)^4}_{\text{G. C. F.}}(2(2x-7)+5x)$

$= 2x^3(2x-7)^4(9x-14)$ {"book" ANSwer}

use Quotient Rule:

④ $\frac{dy}{dx}\frac{4x+3}{(2x-1)^{\frac{1}{2}}} = \frac{4(2x-1)^{\frac{1}{2}} - (4x+3)\cdot\frac{1}{2}(2x-1)^{-\frac{1}{2}}\cdot 2}{((2x-1)^{\frac{1}{2}})^2}$

simplify: $\boxed{\frac{4(2x-1)^{\frac{1}{2}} - (4x+3)(2x-1)^{-\frac{1}{2}}}{(2x-1)}}$

divide each term by $(2x-1)$: $\boxed{4(2x-1)^{-\frac{1}{2}} - (4x+3)(2x-1)^{-\frac{3}{2}}}$ Finished per exam

"book" ANSwer: factor completely

$= (2x-1)^{-\frac{3}{2}}(4(2x-1)-(4x+3)) = (2x-1)^{-\frac{3}{2}}(8x-4-4x-3)$

$= (2x-1)^{-\frac{3}{2}}(4x-7) = \boxed{\frac{(4x-7)}{\sqrt{(2x-1)^3}}}$

⑨ $\frac{dy}{dx}$ $\frac{x}{1-2x}$ use quotient rule

$$= \frac{1(1-2x) - x(-2)}{(1-2x)^2} = \frac{1-2x+2x}{(1-2x)^2} = \boxed{\frac{1}{(1-2x)^2}}$$

⑩ Expand $(2x-3y)^4$ {easy to use Pascal's Triangle}

$$\boxed{\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \end{array}}$$

$$(2x)^4 + 4(2x)^3(-3y)^1 + 6(2x)^2(-3y)^2 + 4(2x)^1(-3y)^3 + (-3y)^4 =$$

$$16x^4 + (32x^3)(-3y) + (24x^2)(9y^2) + (8x)(-27y^3) + 81y^4 =$$

$$\boxed{16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4}$$

⑪ find the first 4 terms : $(a+1)^{15}$

$$\sum_{r=0}^{r=3} \binom{15}{r} a^{15-r} (1)^r$$

$$= \binom{15}{0} a^{15}(1)^0 + \binom{15}{1} a^{14}(1)^1 + \binom{15}{2} a^{13}(1)^2 + \binom{15}{3} a^{12}(1)^3$$

$$= \boxed{a^{15} + 15a^{14} + 105a^{13} + 455a^{12}}$$

⑫ $\left(x^3 + \frac{2}{x}\right)^7$ find the coefficient of x^5 term

General term:

$$\binom{7}{r}(x^3)^{7-r}\left(\frac{2}{x}\right)^r$$ {simplify to isolate the x term}

$$= \binom{7}{r} x^{21-3r} \cdot 2^r \cdot x^{-r} = \binom{7}{r} 2^r x^{21-3r-r}$$

$$= \binom{7}{r} 2^r x^{21-4r}$$ then $21-4r=5$ so $\boxed{r=4}$

then replace $R=4$ in the general term:

$$\binom{7}{4} 2^4 \cdot x^5 = 560x^5$$ $\boxed{\text{answer: } 560}$

13. $\left(x^2 - \frac{1}{2x}\right)^{18}$ find the term independent of x.
{i.e. where the power of x is 0, $x^0 = 1$}
The constant term

General Term:

$$\binom{18}{r}(x^2)^{18-r}\left(-\frac{1}{2x}\right)^r$$ {simplify by combining like terms.}

$$\binom{18}{r}\left(x^{36-2r}\right)(-1)^r(2)^{-r}(x)^{-r}$$

$$= \binom{18}{r}\left(-\frac{1}{2}\right)^r x^{36-3r} \rightarrow \text{then } 36 - 3r = 0$$

$\boxed{r = 12}$

{Replace r = 12}

use calculator in maths mode

$$\binom{18}{12}\left(-\frac{1}{2}\right)^{12} x^0 = 18564 \cdot \frac{1}{4096} = \boxed{\frac{4641}{1024}}$$

14. Evaluate $(1.1)^5$ using binomial expansion

$$\sum_{r=0}^{r=5} \binom{5}{r}(1)^{5-r}\left(\frac{1}{10}\right)^r$$

$$= \binom{5}{0}(1)^5\left(\frac{1}{10}\right)^0 + \binom{5}{1}(1)^4\left(\frac{1}{10}\right)^1 + \binom{5}{2}(1)^3\left(\frac{1}{10}\right)^2$$

$$+ \binom{5}{3}(1)^2\left(\frac{1}{10}\right)^3 + \binom{5}{4}(1)^1\left(\frac{1}{10}\right)^4 + \binom{5}{5}(1)^0\left(\frac{1}{10}\right)^5$$

$$= 1 + \frac{5}{10} + \frac{10}{100} + \frac{10}{1,000} + \frac{5}{10,000} + \frac{1}{100,000}$$

$$= 1 + 0.5 + 0.1 + 0.01 + 0.0005 + 0.00001$$

$$= \boxed{1.61051}$$

Part 2 Differentiate 1-5

① $\frac{dy}{dx}\, 9e^{1-x^2} = 9e^{1-x^2}(-2x)$ {chain rule}

$= \boxed{-18x\, e^{1-x^2}}$ {simplified}

② $\frac{dy}{dx}\, \frac{e^{2x}+4}{x^3}$ { quotient rule }

$= \frac{(2e^{2x})(x^3) - (e^{2x}+4)(3x^2)}{x^6}$ ← simplify

$= \frac{2x^3e^{2x} - 3x^2e^{2x} - 12x^2}{x^6}$ { x^2 can be canceled in each term }

$= \boxed{\frac{2xe^{2x} - 3e^{2x} - 12}{x^4}}$ ← { finished if on an exam. }

to get "book" answer, factor out e^{2x} from the first 2 terms.

$= \boxed{\frac{e^{2x}(2x-3)-12}{x^4}}$

③ $\frac{dy}{dx}\, \ln\left(\frac{5x}{2x-3}\right)$ { remember that the derivative of $\ln x$ is the reciprocal of the argument, i.e. $\frac{1}{x}$ }

use chain rule: use quotient rule.

$= \left(\frac{2x-3}{5x}\right)\left(\frac{5(2x-3)-5x(2)}{(2x-3)^2}\right)$ {simplify}

$\left(\frac{(2x-3)}{(5x)}\right)\left(\frac{-15}{(2x-3)^2}\right) = \frac{-15(2x-3)}{5x(2x-3)^2} = \boxed{\frac{-3}{x(2x-3)}}$

④ $\frac{dy}{dx}\, \ln(1+\sin x) = \left(\frac{1}{1+\sin x}\right)\cos x = \boxed{\frac{\cos x}{1+\sin x}}$

5) $\frac{dy}{dx}$ $\tan(x^2+3)$ chain rule sec⇐sec⇐tan

$$= \sec^2(x^2+3)(2x) = \boxed{2x\sec^2(x^2+3)}$$

6) Express as Partial Fractions

a) $$\frac{2x-1}{(2x+1)(x-3)} = \frac{A}{(2x+1)} + \frac{B}{(x-3)}$$

$$2x-1 = A(x-3) + B(2x+1)$$

let $x = -\frac{1}{2}$: $2(-\frac{1}{2})-1 = A(-\frac{1}{2}-3)$

$$-2 = -\frac{7}{2}A \qquad \boxed{A = \frac{4}{7}}$$

let $x = 3$:

$$2(3)-1 = B(7) \qquad \boxed{B = \frac{5}{7}}$$

$$\boxed{\text{so } \frac{2x-1}{(2x+1)(x-3)} = \frac{4}{7(2x+1)} + \frac{5}{7(x-3)}}$$

(b) $$\frac{3x^2-4}{x(x^2+1)} = \frac{A}{x} + \frac{B}{x^2+1}$$

$$3x^2-4 = A(x^2+1) + Bx$$

let $x=0$: $-4 = A$ $\qquad \boxed{A=-4}$

{replace $A=-4$ then solve for B}

$$3x^2-4 = -4(x^2+1) + Bx$$

$$3x^2\cancel{-4} = -4x^2\cancel{-4} + Bx$$

$$7x^2 = B\cancel{x}$$ {divide both sides by x}

$$7x = B \qquad \boxed{B=7x}$$

so $$\boxed{\frac{3x^2-4}{x(x^2+1)} = \frac{-4}{x} + \frac{7x}{x^2+1}}$$

⑥ (c) $\dfrac{2}{(x-1)^2(x+1)} = \dfrac{A}{(x+1)} + \dfrac{B}{(x-1)} + \dfrac{C}{(x-1)^2}$

$$2 = A(x-1)^2 + B(x-1)(x+1) + C(x+1)$$

let $x=1$: $2 = C(2) \longrightarrow \boxed{C=1}$

let $x=-1$: $2 = 4A \longrightarrow \boxed{A=\frac{1}{2}}$

let $x=0$ and replace $A=\frac{1}{2}$, $C=1$

$2 = \frac{1}{2}(-1)^2 + B(-1)(1) + 1(1)$

$2 = \frac{1}{2} - B + 1 \longrightarrow \boxed{B=-\frac{1}{2}}$

so $\boxed{\dfrac{2}{(x-1)^2(x+1)} = \dfrac{1}{2(x+1)} - \dfrac{1}{2(x-1)} + \dfrac{1}{(x-1)^2}}$

(d) $\dfrac{2x-1}{(x-2)(x+1)(x+3)} = \dfrac{A}{(x-2)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+3)}$

$$2x-1 = A(x+1)(x+3) + B(x-2)(x+3) + C(x-2)(x+1)$$

let $x=2$: $2(2)-1 = A(3)(5) \longrightarrow \boxed{A=\frac{1}{5}}$

let $x=-1$: $2(-1)-1 = B(-3)(2) \longrightarrow \boxed{B=\frac{1}{2}}$

let $x=-3$: $2(-3)-1 = C(-5)(-2) \longrightarrow \boxed{C=\frac{7}{10}}$

$\boxed{\text{so } \dfrac{2x-1}{(x-2)(x+1)(x+3)} = \dfrac{1}{5(x-2)} + \dfrac{1}{2(x+1)} + \dfrac{7}{10(x+3)}}$

(difference of perfect squares)

(e) $\dfrac{4x-1}{x^2(x^2-4)} = \dfrac{4x-1}{x^2(x-2)(x+2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-2} + \dfrac{D}{x+2}$

$$4x-1 = A(x)(x-2)(x+2) + B(x-2)(x+2) + C(x^2)(x+2) + D(x^2)(x-2)$$

let $x=0$: $4(0)-1 = B(-2)(2) \longrightarrow \boxed{B=\frac{1}{4}}$

let $x=2$: $4(2)-1 = C(4)(4) \longrightarrow \boxed{C=\frac{7}{16}}$

let $x=-2$: $4(-2)-1 = D(-2)^2(-4) \longrightarrow \boxed{D=\frac{9}{16}}$

let $x=1$ replace $B=\frac{1}{4}$, $C=\frac{7}{16}$, $D=\frac{9}{16}$ * $x=1$ is arbitrary, choose ANY value.

$4(1)-1 = A(1)(-1)(3) + \frac{1}{4}(-1)(3) + \frac{7}{16}(1^2)(3) + \frac{9}{16}(+1)^2(-1)$

$3 = -3A - \frac{3}{4} + \frac{21}{16} - \frac{9}{16} \longrightarrow \boxed{A=-1}$

so $\boxed{\dfrac{4x-1}{x^2(x^2-4)} = -\dfrac{1}{x} + \dfrac{1}{4x^2} + \dfrac{7}{16(x-2)} + \dfrac{9}{16(x+2)}}$

(7) simplify $\dfrac{3x^3 - 2x + 4}{x - 4}$

$$\begin{array}{r l}
 & 3x^2 + 12x + 46 \\
x - 4 \,\big) & 3x^3 - 0x^2 - 2x + 4 \\
 & -(3x^2 - 12x^2) \\
 & 12x^2 - 2x \\
 & -(12x^2 - 48x) \\
 & 46x + 4 \\
 & -(46x - 184) \\
 & 188
\end{array}$$

answer: $\boxed{3x^2 + 12x + 46 + \dfrac{188}{x-4}}$

(8)

$$\begin{array}{r l}
 & x + 1 \quad + \boxed{\dfrac{2}{x^2 - x - 2}} \\
x^2 - x - 2 \,\big) & x^3 - 0x^2 - 3x + 0 \\
 & -(x^3 - x^2 - 2x) \\
 & x^2 - x + 0 \\
 & -(x^2 - x - 2) \\
 & 2
\end{array}$$

Factorise denominator

then $\dfrac{2}{x^2 - x - 2} = \dfrac{2}{(x-2)(x+1)} = \dfrac{A}{(x-2)} + \dfrac{B}{(x+1)}$

$$2 = A(x+1) + B(x-2)$$

let $x = 2$: $2 = 3A \rightarrow \boxed{A = \frac{2}{3}}$

let $x = -1$: $2 = -3B \rightarrow \boxed{B = -\frac{2}{3}}$

so $\dfrac{2}{x^2 - x - 2} = \dfrac{2}{3(x-2)} - \dfrac{2}{3(x+1)}$

answer: $\boxed{x + 1 + \dfrac{2}{3(x-2)} - \dfrac{2}{3(x+1)}}$

Part 3

① $\frac{dy}{dx}$ $x^3(3-x^4)^5$ product Rule

$= 3x^2(3-x^4)^5 + x^3 \cdot 5(3-x^4)^4(-4x^3)$ simplify

$= \boxed{3x^2(3-x^4)^5 - 20x^6(3-x^4)^4}$ finished if on exam.

factorise to get "book" answer.

$= \underbrace{x^2(3-x^4)^4}_{\text{G.C.F.}}(3(3-x^4) - 20x^4)$

$= x^2(3-x^4)^4(9-3x^4-20x^4)$

$= \boxed{x^2(3-x^4)^4(9-23x^4)}$

② $\frac{dy}{dx}$ $\frac{(6x+5)^2}{(2x-3)^3}$ use quotient Rule

$= \frac{2(6x+5)(6)(2x-3)^3 - (6x+5)^2 \cdot 3(2x-3)^2(2)}{((2x-3)^3)^2}$

$= \frac{12(6x+5)(2x-3)^3 - 6(6x+5)^2(2x-3)^2}{(2x-3)^6}$

* DIVIDE EACH TERM by $(2x-3)^2$:

$= \boxed{\frac{12(6x+5)(2x-3) - 6(6x+5)^2}{(2x-3)^4}}$

To get "book" answer, factorise:

$= \frac{6(6x+5)(2(2x-3)-(6x+5))}{(2x-3)^4}$

$= \frac{6(6x+5)(4x-6-6x-5)}{(2x-3)^4}$

$= \boxed{\frac{6(6x+5)(-2x-11)}{(2x-3)^4}}$

(3) $e^{\cos^2 x} = e^{(\cos x)^2}$ chain rule

$$\frac{dy}{dx} = e^{\cos^2 x}(2\cos x)(-\sin x) \quad \{\text{rearrange}\}$$

$$= \boxed{-2\cos x \sin x\, e^{\cos^2 x}}$$

(4) $\frac{dy}{dx}\ 4x^3 \ln x$ use product rule.

answer:

$$= 12x^2 \ln x + 4x^3 \cdot \frac{1}{x} = \boxed{12x^2 \ln x + 4x^2}$$

to get "book" aswer, factorise!

$$= \boxed{4x^2(3\ln x + 1)}$$

(5) $\frac{dy}{dx}\ \exp(x+\sin x) = \frac{dy}{dx}\ e^{x+\sin x}$ (chain rule)

$$= \boxed{e^{x+\sin x}(1+\cos x)} \quad \underline{\text{OR}}\ \exp(x+\sin x)(1+\cos x)$$

(6) $\frac{2x^3 - x + 7}{4-x}$ | (a)

$$\begin{array}{r} -2x^2 - 8x - 31 + \frac{131}{-x+4} \\ -x+4 \overline{)\,2x^3 + 0x^2 - x + 7} \\ -(2x^3 - 8x^2) \\ \hline 8x^2 - x \\ -(8x^2 - 32x) \\ \hline 31x + 7 \\ -(31x - 124) \\ \hline 131 \end{array}$$

(a) $\boxed{\text{answer: } -2x^2 - 8x - 31 + \frac{131}{4-x}}$

(b) $\frac{dy}{dx}\ -2x^2 - 8x - 31 + \frac{131}{4-x}$

$$= -4x - 8 + \left[-131(4-x)^{-2}(-1)\right]$$

$$= \boxed{-4x - 8 + \frac{131}{(4-x)^2}}$$