Higher Mathematics - Practice Examination A

Please note … the format of this practice examination is different from the current format. The paper timings are different and calculators can be used throughout.

MATHEMATICS

Higher Grade - Paper I

Time allowed - 2 hours

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. Calculators may be used.
3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y \ + 2gx + 2fy + c = 0$ represents a circle centre $(-g\ ,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(\,a\ ,b\,)$ and radius r.

Scalar Product: $a \centerdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \centerdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad where \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad and \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned}
\sin\left(A \pm B\right) &= \sin A \cos B \pm \cos A \sin B \\
\cos\left(A \pm B\right) &= \cos A \cos B \mp \sin A \sin B \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A \\
\sin 2A &= 2\sin A \cos A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be attempted

1. PQRS is a rhombus. Vertices P , Q and S have coordinates (-5 , -4) , (-2 , 3) and (2 , -1) respectively.

Establish the coordinates of the fourth vertex R and hence, or otherwise, find the equation of the diagonal PR. **(4)**

2. A circle has as its equation $x^2 + y^2 - 4x + 3y + 5 = 0$.

Find the equation of the tangent at the point (3,-2) on the circle. **(5)**

3. Find $f'(x)$ when $f(x) = \dfrac{x^3 - 6\sqrt{x}}{x^2}$. **(4)**

4. A function is given as $g(x) = \dfrac{2}{x}$.

(a) State a suitable domain for this function on the set of real numbers. **(1)**

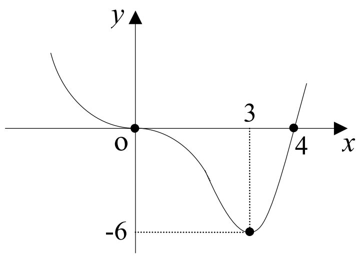
(b) Evaluate $g(g(\frac{1}{2}))$. **(2)**

(c) Find a formula for $g(g(x))$ in its simplest form. **(3)**

5. Given that $\displaystyle\int_1^a (5 - 2x)\, dx = 2$.

Find **algebraically** the two possible values of a . **(4)**

6. The diagram below shows the graph of $y = f(x)$.



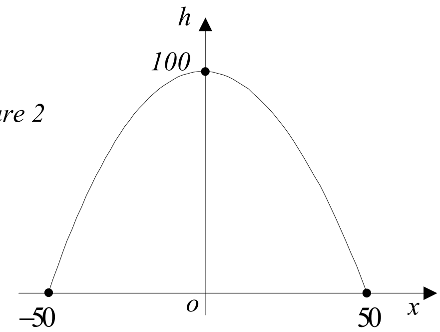
Make a rough sketch of the graph of $y = f(-x) + 6$. **(4)**

7. Given that $x - 1$ is a factor of $x^3 + kx^2 - 5x + 6$, find the value of k and hence fully factorise the expression. **(4)**

8. The famous Gateway Arch in the United States is parabolic in shape. *Figure 2* shows a rough sketch of the arch relative to a set of rectangular axes.

Figure 1

Figure 2



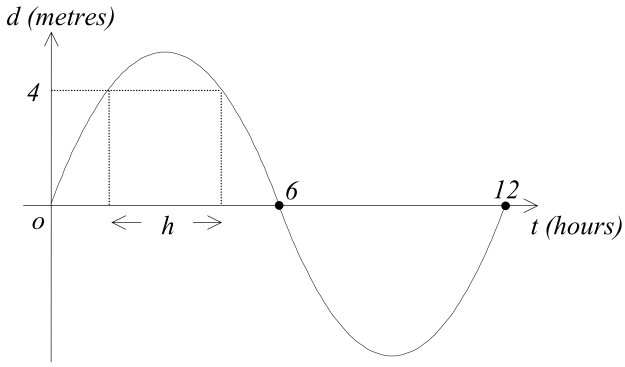
From *figure 2* establish the equation connecting h and x . **(3)**

9. Given that the points (3 , -2) , (4 , 5) and (-1 , a) are collinear , find the value of a . **(3)**

10. A and B are the points (-2,-1,4) and (3,4,-1) respectively.

Find the coordinates of the point C given that $\dfrac{AC}{CB} = \dfrac{3}{2}$. **(5)**

11. The diagram below shows the depth of water in a small harbour during a standard 12-hour cycle.



It has been found that the depth of water follows the relationship $d = 6\sin 30t$, where d is the depth in metres (above or below a mean height) and t is the time elapsed in hours from the start of the cycle.

Calculate the value of h , the length of time, in hours and minutes, that the depth in the harbour is greater than or equal to 4 metres.
Give your answer to the nearest minute. **(4)**

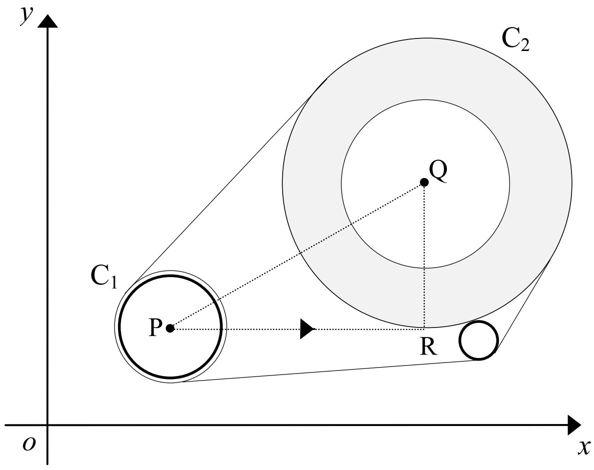
12. Find the derivative , with respect to x , of $\frac{1}{6}(1+2x)^3 + \sin 2x$. **(4)**

13. Solve algebraically the equation

$$3\cos 2x^\circ - \cos x^\circ - 2 = 0\ , \qquad 0 \le x < 360.$$ **(6)**

14. The diagram opposite represents part of a belt driven pulley system for a compact disc turntable .

P and Q are the centres of the circles C_1 and C_2 respectively. **PQ = 6 units** .
PR is parallel to the *x*-axis and is a tangent to circle C_2
The coordinates of P are $(2 \cdot 8\ ,\ 2)$.



(a) Given that angle QPR = 30° , establish the coordinates of the point Q , rounding any calculations to one decimal place where necessary. **(4)**

(b) Hence write down the equation of circle C_2 . **(2)**

15. A radioactive substance decays according to the formula $M_t = M_o\, 8^{-0 \cdot 3t}$, where M_o is the intitial mass of the substance, M_t is the mass remaining after t years.

Calculate , to the nearest day, how long a sample would take to **half** its original mass. **(5)**

16. A sequence of numbers is defined by the recurrence relation $U_{n+1} = kU_n + c$, where k and c are constants.

(a) Given that $U_0 = 10$, $U_1 = 14$ and $U_2 = 17 \cdot 2$, find **algebraically** , the values of k and c . **(3)**

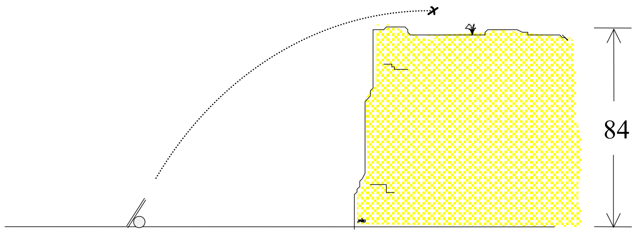
(b) Calculate the value of E given that $E = \frac{3}{4}(L - 2)$, where L is the limit of this sequence. **(3)**

17. A function is given as $f(\theta) = \sin^2\theta + 2\sin\theta - 3$ for $0 \leq \theta \leq 2\pi$.

(a) Express the function in the form $f(\theta) = (\sin\theta + a)^2 + b$ and write down the values of a and b . **(3)**

(b) Hence, or otherwise, state the minimum value of this function and the corresponding replacement for θ. **(2)**

18. A group of soldiers decide the best way to scale a cliff is to fire a metal hook, with a rope attached, over the top of the cliff and hope it catches on to something solid.



It is known that the firing device will launch the rope in such a way that the height , H feet , above the ground is given by

$$H(d) = d - \frac{d^2}{330} ,$$

where d feet is the **horizontal** distance travelled.

Given that the height of the cliff is 84 feet , will this device be able to throw the rope high enough ? Justify your answer with the appropriate working. **(6)**

[END OF QUESTION PAPER]

Higher Mathematics Practice Exam A **Marking Scheme - Paper 1**

1. For establishing coordinates R(5 , 6) .... (2)
For gradient of PR , $m_{pr} = 1$ .... (1)
Then for equation $y = x + 1$ .... (1) **[4 marks]**

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2. For establishing the centre $C(2, -\frac{3}{2})$ .... (1)
For finding the gradient of the radius $m_r = -\frac{1}{2}$ .... (1)
For gradient of tangent $m_{tan} = 2$ .... (1)
For using $y - b = m(x - a)$, or equivalent, with m = 2 and (3,-2) .... (1)
For equation $y = 2x - 8$ .... (1)

[5 marks]

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3. For $f(x) = x^{-2}(x^3 - 6x^{\frac{1}{2}})$ .... (1)
$= x - 6x^{-\frac{3}{2}}$ .... (2) (1 for each part)
$f'(x) = 1 + 9x^{-\frac{5}{2}}$ .... (1) **[4 marks]**

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4. (a) For $x \neq 0$ .... (1) **[1 mark]**
(b) For $g(\frac{1}{2}) = \dfrac{2}{\frac{1}{2}} = 4$ (or equivalent) .... (1)
$g(g(\frac{1}{2})) = \dfrac{2}{4} = \dfrac{1}{2}$ .... (1) **[2 marks]**

(c) For $g(g(x)) = \dfrac{2}{\frac{2}{x}}$ .... (1)
$= \dfrac{2x}{2}$ .... (1)
$= x$ .... (1) **[3 marks]**

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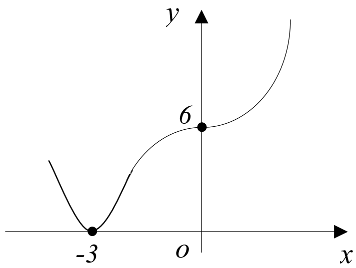
5. For $\left[5x - x^2\right]_1^a = 2$ .... (1)
$\left(5a - a^2\right) - \left(5 - 1\right)$ .... (1)
$a^2 - 5a + 6 = 0$ .... (1)
For answer $a = 2$ or $a = 3$ .... (1) **[4 marks]**

6. For reflecting in y-axis .... (1)
For translation .... (1)
For annotation -3 & 6 .... (2)

[4 marks]



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7. For deciding on method (synthetic division) and dividing by 1 .... (1)

For	1	1	k	-5	6
			1	k+1	k-4
		1	k+1	k-4	k+2

.... (1)

For $k + 2 = 0 \quad \therefore \quad k = -2$ .... (1)
(For $f(1) = 0$ etc., full marks if k correct)
For answer (x - 1)(x - 3)(x + 2) .... (1) **[4 marks]**

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8. For $h = k\,(2500 - x^2)$ (or equiv.) .... (1)
For realising that at $x = 0 \quad h = 2500 \times k = 100$ .... (1)
For $k = \frac{1}{25}$ .... (1)
$\therefore \quad h = \frac{1}{25}\left(2500 - x^2\right)$ (no marks off for not stating final equ.) **[3 marks]**

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9. Deciding on correct method .... (1)
For $\dfrac{5+2}{4-3} = \dfrac{a-5}{-1-4}$ (or equiv.) .... (1)
For answer $a = -30$ .... (1) **[3 marks]**

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10. For $2\overrightarrow{AC} = 3\overrightarrow{CB}$ .... (1)
For $2\left(\underset{\sim}{c} - \underset{\sim}{a}\right) = 3\left(\underset{\sim}{b} - \underset{\sim}{c}\right)$ .... (1)
$5\underset{\sim}{c} = 3\underset{\sim}{b} + 2\underset{\sim}{a}$ .... (1)
For substituting components .... (1)
For answer C(1 , 2 , 1) .... (1)
* (no marks off if left in component form) **[5 marks]**

11. For $6\sin 30t = 4$ .... (1)
For $30t = 41\cdot 8^\circ$ and $30t = 138\cdot 2^\circ$ .... (1)
For t = 1 hour 24 min. and t = 4 hours 36 min. .... (1)
For answer h = 3 hours 12 min. .... (1) **[4 marks]**

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12. For $\frac{1}{2}(1+2x)^2 \,.\, 2 \;+\; 2\cos 2x$ .... (4)
(split as follows $\frac{1}{2}(1+2x)^2$; $\times 2$; 2 ; $\cos 2x$ ...... 1 *mark each*) **[4 marks]**

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13. For $3(2\cos^2 x - 1) - \cos x - 2 = 0$ .... (1)
$6\cos^2 x - \cos x - 5 = 0$ .... (1)
For $\cos x = -\frac{5}{6}$ or $\cos x = 1$ .... (1)
Then $x =$ 0 , $146\cdot 4$, $213\cdot 6$ (1 each) .... (3)
(no marks off for 360) **[6 marks]**

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14. (a) For correct strategy e.g. using trig. .... (1)
For PR = 5 • 2 .... (1) and QR = 3 .... (1)
For coordinates Q(8,5) .... (1)
[4 marks]

(b) For establishing that $r = 3$ .... (1)
For equation $(x-8)^2 + (y-5)^2 = 9$ .... (1)
[2 marks]

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15. For realising $8^{-0\cdot 3t} = 0\cdot 5$ .... (1)
For taking logs $\log 8^{-0\cdot 3t} = \log 0\cdot 5$ .... (1)
Then $-0\cdot 3t\log 8 = \log 0\cdot 5$ .... (1)
$-0\cdot 3t = \dfrac{\log 0\cdot 5}{\log 8}$ .... (1)
For answer t = 406 days .... (1) **[5 marks]**

i) no marks off if pupils assume an initial mass.
ii) accept 405 or other answer if pupil has rounded early.

16. (a) For the system $\left.\begin{array}{r} 14 = 10k + c \\ 17\cdot 2 = 14k + c \end{array}\right\}$ .... (1)

For $k = 0\cdot 8$ *and* $c = 6$ (1 each) .... (2) **[3 marks]**

(b) For knowing $L = \dfrac{b}{1-a}$ .... (1)

$\therefore \quad L = \dfrac{6}{1-0\cdot 8} = 30$ .... (1)

Then answer $E = 21$ .... (1) **[3 marks]**

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17. (a) For $\left[\ (\sin\theta + 1)^2 - 1\right] - 3$ .... (1)

$\left(\sin\theta + 1\right)^2 - 4$ .... (1)

$\therefore \quad a = 1$ *and* $b = -4$ .... (1) **[3 marks]**

(No marks off if a and b are not implicitly or explicitly stated.)

(b) Minimum value of -4 .... (1)

@ $\theta = \dfrac{3\pi}{2}$ .... (1) **[2 marks]**

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18. Pupil adopts strategy to locate maximum T.P. .... (1)

For finding derivative $H'(d) = 1 - \dfrac{d}{165}$ .... (1)

For equating derivative to zero .... (1)

Then for value of d at turning point $d = 165$ feet .... (1)

For evaluating $H(165) = 82.5$ feet .... (1)

Rope **will not** be thrown high enough .... (1) **[6 marks]**

Total	**84 marks**

Higher Mathematics - Practice Examination A

Please note … the format of this practice examination is different from the current format. The paper timings are different and calculators can be used throughout.

MATHEMATICS

Higher Grade - Paper II

Time allowed - 2 hours 30 minutes

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. Calculators may be used.
3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

The equation $x^2 + y \ + 2gx + 2fy + c = 0$ represents a circle centre $(-g\,,-f)$ and radius $\sqrt{\left(g^2 + f^2 - c\right)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(\,a\,,b\,)$ and radius r.

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \textit{where} \quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \textit{and} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric formulae:

$$\begin{aligned}
\sin\left(A \pm B\right) &= \sin A \cos B \pm \cos A \sin B \\
\cos\left(A \pm B\right) &= \cos A \cos B \mp \sin A \sin B \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A \\
\sin 2A &= 2\sin A \cos A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

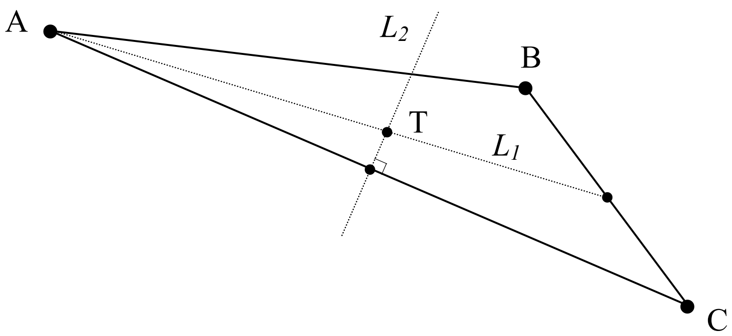
$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

All questions should be attempted

1. Triangle ABC has as its vertices A(-18,6) , B(2,4) and C(10,-8) .

L_1 is the median from A to BC.

L_2 is the perpendicular bisector of side AC.



(a) Find the equations of L_1 and L_2 . **(7)**

(b) Hence find the coordinates of T . **(3)**

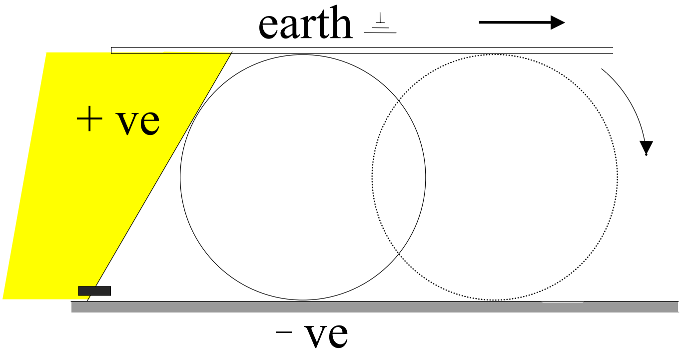
2. Two functions are defined as $f(x) = px^2 - 1$ and $h(x) = \dfrac{5x + q}{2}$,
where p and q are constants.

(a) Given that $f(2) = h(2) = 7$, find the values of p and q . **(3)**

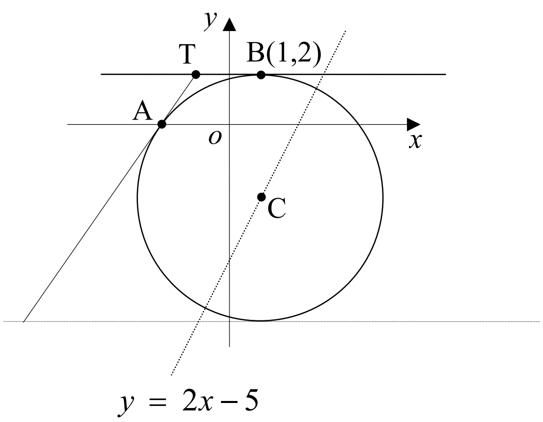
(b) Show that $h\big(f(x)\big) = \frac{1}{2}(10x^2 - 1)$. **(2)**

(c) Find the value of the constant k when $2\big[h\big(f(x)\big)\big] - 4 = k\big[f(x)\big]$. **(3)**

3. The diagram below is a sketch of a roller safety switch. If the device is tilted clockwise the small cylinder will roll to the right thus breaking the circuit.



A schematic of the device is shown below with a pair of rectangular axes added.
The equation of the dotted line through the centre of the circle, C , is $y = 2x - 5$.
B has coordinates (1,2) as shown.
The line through the points A and T is a tangent to the circle at A.

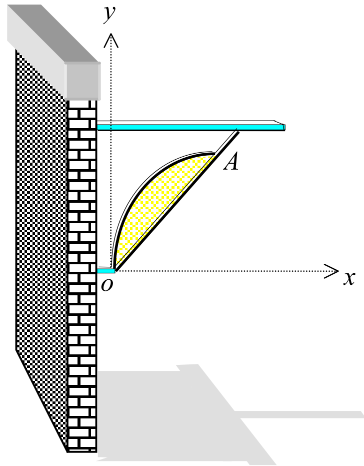


(a) Establish the equation of the circle. **(4)**

(b) Hence find the coordinates of point A. **(3)**

(c) Find the equation of the line AT and the coordinates of T. **(6)**

4. The diagram opposite shows part of a wrought iron plant pot hanger attached to a wall .

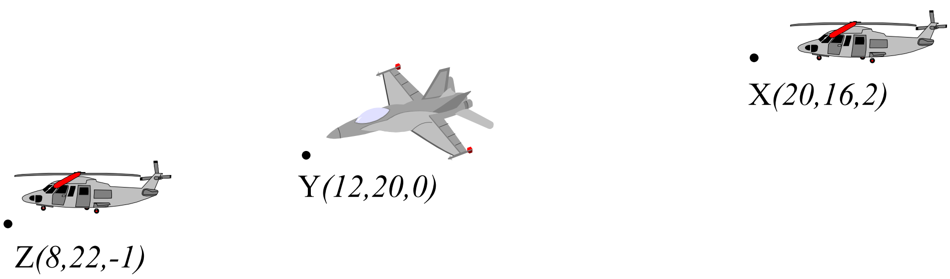


Relative to the axes drawn the line OA has as its equation $y = x$ and the curve from O to A has as its equation $y = \frac{1}{11}(22x - x^2)$.

(a) Establish the coordinates of the point A . **(3)**

(b) The shaded area between the curve and the line represents a small decorative wooden plaque . Given that the units are in inches, calculate the area of the wooden plaque to the nearest square inch. **(5)**

5. Three military aircraft are on a joint training mission . Their positions relative to each other, within a three dimensional framework, are shown in the diagram below :

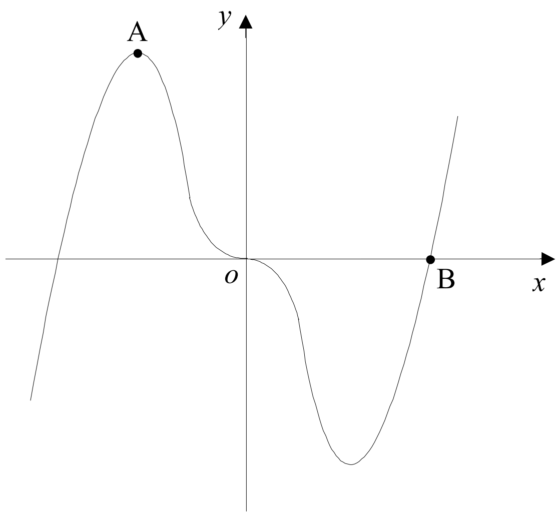


(a) Show that the three aircraft are collinear . **(3)**

(b) Given that the actual distance between Z and Y is 42km , how far away from Z is X ? **(1)**

(c) Following further instructions aircraft Y moves to a new position (50,15,-8) . The other two aircraft remain where they are . For this new situation , calculate the size of $\angle XYZ$. **(5)**

6. The curve shown below has as its equation $y = 3x^5 - 5x^3$.

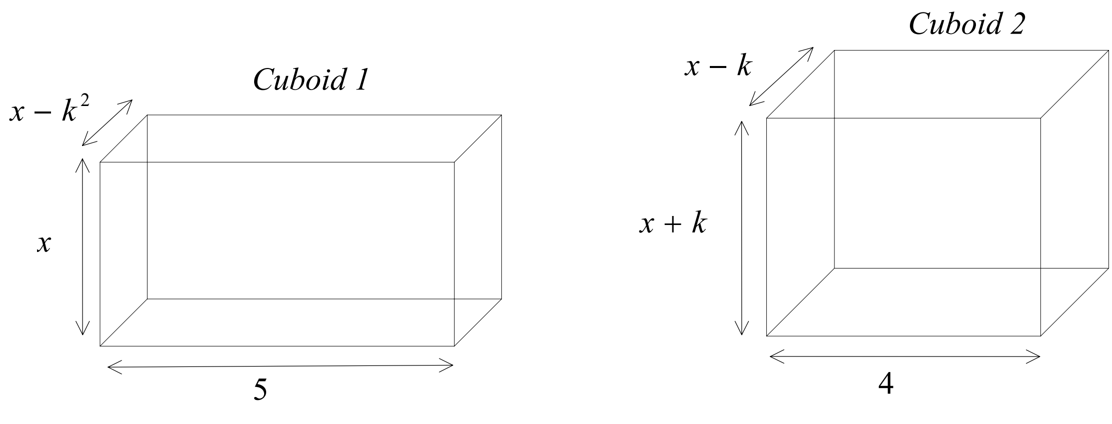


Find algebraically the coordinates of the points A and B . **(7)**

7. The two cuboids below have the same volume, $V_1 = V_2$.

Cuboid 1 has dimensions 5 by x by $x - k^2$ and *Cuboid 2* 4 by $x + k$ by $x - k$ as shown.

All lengths are in centimetres.



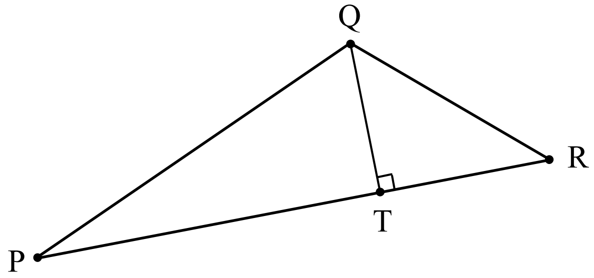
(a) By writing down expressions for the two volumes, V_1 *and* V_2 , and equating them, show that the following equation can be constructed

$$x^2 - 5k^2x + 4k^2 = 0 \ .$$ **(4)**

(b) Given that $k > 0$, find the value of k for which the equation $x^2 - 5k^2x + 4k^2 = 0$ has **equal** roots. **(4)**

(c) Hence solve the equation for x and calculate the volume of each cuboid when k takes this value. **(3)**

8. Triangle PQR , shown below, has vertices P(-4 , -3) , Q(4 , 6) and R(11 , 3).
The altitude has been drawn from Q to PR .

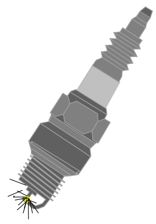


(a) Side PR has as its equation $5y = 2x - 7$. Find the equation of the altitude QT. **(4)**

(b) Hence find the coordinates of T . **(2)**

(c) Calculate the **area** of the triangle in square units. **(4)**

9. Over a period of time the effectiveness of a standard spark plug slowly decreases. It has been found that, in general, a spark plug will loose 8% of its burn efficiency every **two months** while in average use.



(a) A new spark plug is allocated a *Burn Efficiency Rating (BER)* of 120 units. What would the *BER* be for this plug after a year of average use ?
Give your answer correct to one decimal place. **(3)**

(b) After exhaustive research, a new fuel additive was developed.
This additive , when used at the end of every **four month** period, immediately allows the *BER* to increase by 8 units.

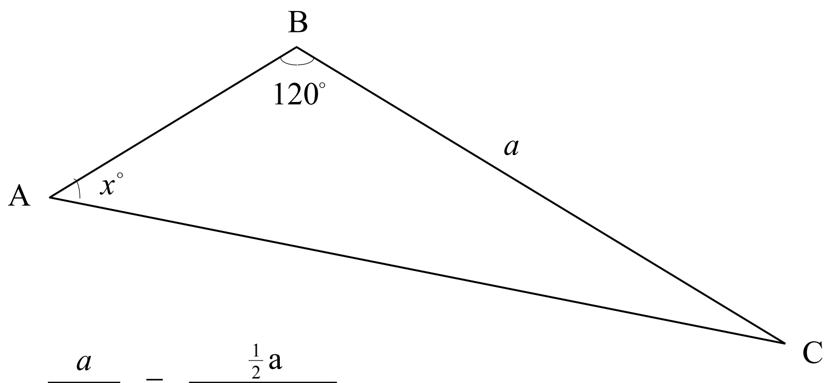
A plug which falls below a *BER* of 93 units should be replaced.

What would be the maximum recommended lifespan for a plug , in months, when using this additive ? **(5)**

(c) The manufacturer is close to developing a new plug which contains a revolutionary double core made from titanium.
Because of its "hot burn" qualities it can operate effectively down to a *BER* of 50 units.
Would this new plug ever need to be replaced if it is used in conjunction with the additive ?
Your answer must be accompanied with the appropriate working. **(3)**

Would it be wise for the manufacturer to make this plug available to the general public ? Explain. **(1)**

10. The diagram below shows triangle ABC in which BC = 2AB .
Angle BAC = x° and angle ABC = 120° .



(a) Show that $\dfrac{a}{\sin x^\circ} = \dfrac{\frac{1}{2}a}{\sin(60 - x)^\circ}$. **(3)**

(b) By simplifying this equation show that $2\sin x^\circ = \sqrt{3}\cos x^\circ$. **(5)**

(c) Hence solve this equation to find the value of x° . **(2)**

[END OF QUESTION PAPER]

Higher Mathematics Practice Exam A — Marking Scheme - Paper 2

1.	(a)	For mid − point of BC , $M(6,-2)$	....	(1)	
		For $m_{AM} = -\frac{1}{3}$	....	(1)	
		Then for equation $L_1 \Rightarrow y = -\frac{1}{3}x$	....	(1)	
		For mid - point of AC , $N(-4,-1)$	....	(1)	
		For $m_{AC} = -\frac{1}{2}$	....	(1)	
		Then for perpendicular $m_{L_2} = 2$	....	(1)	
		Then for equation $L_2 \Rightarrow y = 2x+7$	....	(1)	**[7 marks]**
	(b)	For correct strategy and equating	....	(1)	
		For $x = -3$	....	(1)	
		Then for $y = 1$ giving $T(-3,1)$	....	(1)	**[3 marks]**

2.	(a)	For $f(2) = 4p - 1 = 7$	....	(1)	
		$h(2) = \dfrac{10+q}{2} = 7$	....	(1)	
		Then for $p = 2$ and $q = 4$	....	(1)	**[3 marks]**
	(b)	For $h(f(x)) = \dfrac{5(2x^2-1) + 1}{2}$	....	(1)	
		$= \frac{1}{2}(10x^2 - 1)$ (or equivalent)	....	(1)	**[2 marks]**
	(c)	For $2\left[\frac{1}{2}(10x^2-1)\right] - 4 = k(2x^2-1)$	....	(1)	
		$5(2x^2-1) = k(2x^2-1)$	....	(1) (or equiv.)	
		$\therefore\ k = 5$	....	(1)	**[3 marks]**

3.	(a)	For knowing to substitute $x = 1$ into $y = 2x - 5$	....	(1)	
		For finding y = -3 $\therefore$ centre has coordinates (1,-3)	....	(1)	
		For establishing the radius i.e. $r = 5$	....	(1)	
		For equation $(x-1)^2 + (y+3)^2 = 25$	....	(1)	**[4 marks]**
	(b)	For knowing to sub. $y = 0$ into equation of circle	....	(1)	
		For establishing quadratic and solving to x = -3 or x = 5	....	(1)	
		For choosing -3 and then giving coordinates of A(-3,0)	....	(1)	**[3 marks]**
	(c)	For gradient of radius $m_{CA} = -\frac{3}{4}$	....	(1)	
		For gradient of tangent $m_{AT} = \frac{4}{3}$	....	(1)	
		For using correct point and gradient, m $= \frac{4}{3}$ and (-3,0)	....	(1)	
		Equation of tangent $3y = 4x + 12$	....	(1)	
		For knowing the y-coordinate of T is 2	....	(1)	
		For substitution and solving to $x = -\frac{3}{2}$ then T$(-\frac{3}{2}, 2)$	....	(1)	**[6 marks]**

4. (a) For $x = \frac{1}{11}\left(22x - x^2\right)$ .... (1)

For solving to $x = 0$ or $x = 11$ .... (1)

Then for answer A(11 , 11) .... (1) **[3 marks]**

(b) For $Area = \int_0^{11} \left(\frac{1}{11}(22x - x^2) - x\right)\, dx$ .... (1)

$= \int_0^{11} \left(x - \frac{1}{11}x^2\right)\, dx$ .... (1)

$= \left[\frac{x^2}{2} - \frac{x^3}{33}\right]_0^{11}$ .... (1)

$= \left[\left(\frac{121}{2} - 40\tfrac{1}{3}\right) - (0)\right]$ .... (1)

For answer = 20 square inches .... (1) **[5 marks]**

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5. (a) For choosing any two displacements, 1 mark each.

i.e. $\overrightarrow{ZY} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$ and $\overrightarrow{YX} = \begin{pmatrix} 8 \\ -4 \\ 2 \end{pmatrix}$ .... (2)

For "since $\overrightarrow{YX} = 2\overrightarrow{ZY}$, then x , y and z are collinear"
(or equivalent) .... (1) **[3 marks]**

(b) For answer 126 km .... (1) **[1 mark]**

(c) For $\overrightarrow{YX} = \begin{pmatrix} -30 \\ 1 \\ 10 \end{pmatrix}$ and $\overrightarrow{YZ} = \begin{pmatrix} -42 \\ 7 \\ 7 \end{pmatrix}$ .... (1)

For formula $\cos y = \dfrac{\overrightarrow{YX} \cdot \overrightarrow{YZ}}{|YX| . |YZ|}$ (or equiv.) .... (1)

For $|YX| = \sqrt{1001}$ and $|YZ| = \sqrt{1862}$ .... (1)

For $\overrightarrow{YX} \cdot \overrightarrow{YZ} = 1337$ .... (1)

Then answer $\angle XYZ = 11 \cdot 7^\circ$ .... (1) **[5 marks]**

6. For A : $\frac{dy}{dx} = 15x^4 - 15x^2$ .... (1)

For $15x^4 - 15x^2 = 0$ .... (1)

For $x = -1$ .... (1)

For y - coordinate @ $x = -1$ then A(-1,2) .... (1)

For B : For $3x^5 - 5x^3 = 0$ .... (1)

For $x = 0$ or $x = \pm\sqrt{\frac{5}{3}}$ .... (1)

Then for B$(0, \sqrt{\frac{5}{3}})$ .... (1) **[7 marks]**

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7. (a) For $V_1 = 5x^2 - 5xk^2$ .... (1)

$V_2 = 4x^2 - 4k^2$ .... (1)

For $5x^2 - 5xk^2 = 4x^2 - 4k^2$ .... (1)

Then $x^2 - 5xk^2 + 4k^2 = 0$ .... (1) **[4 marks]**

(b) For $b^2 - 4ac = 0$ .... (1)

For $a = 1$, $b = -5k^2$ and $c = 4k^2$ .... (1)

For $25k^4 - 16k^2 = 0$ .... (1)

For answer $k = \frac{4}{5}$ .... (1) **[4 marks]**

(c) For $x^2 - 3\cdot 2x + 2\cdot 56 = 0$ .... (1)

$\therefore \quad x = 1\cdot 6$ .... (1)

For each cuboid $V = 7\cdot 68\ cm^3$ .... (1) **[3 marks]**

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8. (a) $m_{PR} = \frac{2}{5} \quad \therefore \quad m_{QT} = -\frac{5}{2}$ .... (2)

For equation $2y + 5x = 32$ .... (2) **[4marks]**

(b) For deciding to solve $\left.\begin{matrix} 2y + 5x = 32 \\ 5y - 2x = -7 \end{matrix}\right\}$ .... (1)

For $y = 1$

For $x = 6$ *then* $T(6,1)$ .... (1) **[2marks]**

(c) Various methods : $|PR| = \sqrt{261}$ .... (1)

$|QT| = \sqrt{29}$ .... (1)

For $Area_\Delta = \frac{1}{2} \times \sqrt{261} \times \sqrt{29}$ .... (1)

$= 43\cdot 5$ *square units* .... (1) **[4 marks]**

9.	(a)	For correct multiplier i.e. $0 \cdot 92$	....	(1)	
		For $BER = (0 \cdot 92)^6 \times 120$	....	(1)	
		$= 72 \cdot 8$ *units*	....	(1)	**[3 marks]**
	(b)	For $U_{n+1} = (0 \cdot 92)^2 \times U_n + 8$ (or equiv.)	....	(1)	
		For setting out calculations	....	(1)	
		For realising to look at lower value (i.e. before the 8 is added)	....	(1)	
		For looking at $(0 \cdot 92)^1$ for a 2 month period after U_2	....	(1)	
		For answer **after 10 months**	....	(1)	**[5 marks]**
	(c)	For knowing formula for limit (or equiv.)	....	(1)	
		For $L = \frac{8}{1-(0 \cdot 92)^2} = 52$ *units*	....	(1)	
		For then lower limit 52 - 8 = 44 " It would need to be replaced but not often"	....	(1)	**[3 marks]**
	(d)	"Sales would perhaps burn out !!! " (or equiv.)	....	(1)	**[1 mark]**

10.	(a)	For stating that AB $= \frac{1}{2}$BC $= \frac{1}{2}a$	....	(1)	
		For giving $\angle C = 180 - (120 + x) = 60 - x$	....	(1)	
		Applying the sine rule to answer	....	(1)	**[3 marks]**
	(b)	For $a\sin(60 - x) = \frac{1}{2}a\sin x$	....	(1)	
		$2\sin(60 - x) = \sin x$	....	(1)	
		$2\left[\sin 60 \cos x - \cos 60 \sin x\right] = \sin x$	....	(1)	
		For putting in exact values	....	(1)	
		$\sqrt{3}\cos x = 2\sin x$	....	(1)	**[5 marks]**
	(c)	For $\tan x = \frac{\sqrt{3}}{2}$	....	(1)	
		For answer $x = 40 \cdot 9^\circ$	....	(1)	**[2 marks]**

Total 98 marks