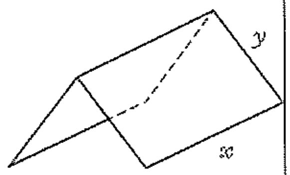
7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is 24 m^2.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$ 3

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame. 7

(a) $A = 2xy$

$2xy = 24$ so $\boxed{y = \frac{12}{x}}$ ✓

$\boxed{\text{LENGTH} = 3x + 4y}$ ✓ {substitute $y = \frac{12}{x}$}

$\text{LENGTH} = 3x + 4\left(\frac{12}{x}\right) = 3x + \frac{48}{x}$ ✓

(b)(i) $L(x) = 3x + \frac{48}{x}$

$L(x) = 3x + 48x^{-1}$ ✓

$\boxed{L'(x) = 3 - \frac{48}{x^2}}$ ✓ Let $3 - \frac{48}{x^2} = 0$ ✓

must check if $x=4$ min or max $3x^2 - 48 = 0$ then $\boxed{x=4}$ ✓

$L''(x) = \frac{96}{x^3}$ | $L''(4) = $ positive $\therefore$ $x=4$ is a minimom ✓

(ii) cost $= 8.25(3x+4y) = 8.25(3\times4 + 4\times3)$ ✓

$= 8.25(24)$

$= \boxed{£198}$ ✓